

# INSTITUTE AND FACULTY OF ACTUARIES

## EXAMINATION

06 October 2020 (am)

### **Subject CM2B – Financial Mathematics and Loss Reserving Core Principles**

Time allowed: One hour and forty five minutes

<p>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</p>
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If you encounter any issues during the examination please contact the Examination Team on T. 0044 (0) 1865 268 873.

- 1** An insurance company sells an investment-linked product with a term of 10 years. The product attracts a guarantee such that the return in each year will be no less than 3%. The guarantee is valued using simulation techniques.

You have been given 100 simulations of the annual return on the underlying investments for each of the next 10 years.

Consider an investment of €100 at time zero.

- (i) Calculate, for each simulation:
- (a) the value of the investment at maturity, ignoring the guarantee.
  - (b) the value of the investment at maturity, allowing for the guarantee.
  - (c) the value of the guarantee at maturity. [10]
- (ii) Calculate the expected present value of the guarantee determined in part (i), discounted to time zero at an annually compounded rate of 2%. [4]
- (iii) Suggest an alternative approach the insurer could use to value the guarantee, commenting on whether the value is likely to be the same. [4]
- [Total 18]

**2** You have been given the expected return on three securities, 1, 2 and 3, and the variance/covariance matrix for these securities. You have also been given the weights invested in each of these three securities in five portfolios, A to E. The portfolios are made up of only these three securities and no other assets are held.

- (i) Calculate the expected return and variance of the return on each of the five portfolios A to E. [8]
- (ii) (a) Plot the expected returns and variances of returns you calculated in part (i) in E–V space.
- (b) Determine, with reasons, whether any of the five portfolios are inefficient. [10]

Consider a new portfolio, Portfolio F, that invests in a mix of only Security 1 and Security 3. Short-selling is not allowed.

- (iii) (a) Calculate the proportion of the portfolio invested in each of Securities 1 and 3 that minimises the variance of the portfolio return.
- (b) Calculate the expected overall return on Portfolio F.
- (c) Calculate the variance of the overall return on Portfolio F.
- (d) Comment on the variance of the return on Portfolio F in relation to the variances of the return on Securities 1 and 3. [10]

You have been provided with past data on the annual return on the market portfolio and the annual return on Asset X in a market where the assumptions of the Capital Asset Pricing Model hold.

- (iv) Calculate the value of Beta for Asset X based on this historic data. [8]
- [Total 36]

**3** A bank offers three savings products to customers:

- A A cash account paying interest at a guaranteed rate of 4% per annum.
- B An investment product that tracks an equity index.
- C A hybrid product that gives a guaranteed minimum return of 3% per annum, plus half of any excess return on the index above 3% per annum.

[For example, if the return on the equity index is 5% then product C will give a return of  $3\% + \frac{1}{2} \times (5\% - 3\%) = 4\%$ . If the return on the index is below 3% then product C will give a return of 3%.]

You have been given a distribution of the 1-year return on the equity index in the spreadsheet.

An investor plans to invest £1,000 in each of products A, B and C at time  $t = 0$  to be withdrawn after 1 year (at time  $t = 1$ ).

(i) Calculate the mean and variance of the value of each investment at time  $t = 1$ . [10]

(ii) Calculate the downside semi-variance of each investment at time  $t = 1$ . [6]

The investor uses a quadratic utility function  $U(w) = w - 0.0003w^2$ .

(iii) Calculate the expected utility of each investment at time  $t = 1$ . [10]

(iv) Explain why the measures in parts (i), (ii) and (iii) give different answers to which investment might be the best option for this investor. [8]

(v) Plot a chart of the utility at time  $t = 1$  of product B for a range of equity index returns from  $-20\%$  to  $+30\%$ . [4]

(vi) Comment on how your chart in part (v) would differ if the investor's utility function:

(a) exhibited constant relative risk-aversion.

(b) was risk-neutral.

[8]

[Total 46]

**END OF PAPER**