

# **INSTITUTE AND FACULTY OF ACTUARIES**

## **EXAMINATION**

23 April 2021 (am)

### **Subject CM2B - Financial Engineering and Loss Reserving Core Principles**

Time allowed: One hour and forty-five minutes

If you encounter any issues during the examination please contact the Assessment Team on  
T. 0044 (0) 1865 268 873.

- 1** The annual yields,  $i$ , from a fund are independent and identically distributed. Each year, the distribution of  $(1 + i)$  is log-normal with parameters  $\mu = 0.05$  and  $\sigma^2 = 0.004$ .

An amount of \$5,000 is invested in the fund at the start of each of the next 20 years, i.e. at times  $t = 0, t = 1, t = 2, \dots, t = 19$ .

- (i) Calculate, using the probability distribution above, the expected accumulated value of the fund at time  $t = 20$ . [6]

A student has performed a simulation of the annual returns from the distribution and the results are provided in the 'Q1 data' worksheet.

- (ii) Calculate the expected accumulated value of the fund at time  $t = 20$ , using the simulated returns. [3]

- (iii) Comment on the results in parts (i) and (ii). [3]

- (iv) Using the simulated returns from part (ii):

- (a) calculate, the cumulative probability that the fund value at time  $t = 20$  reaches or exceeds each of the values from \$100,000 to \$300,000 as set out on the '1iv' worksheet in increments of \$5,000.

- (b) plot a chart of the cumulative probabilities.

- (c) comment on the shape of the chart in part (iv)(b).

[8]

[Total 20]

- 2 A company decides to give each of its employees a new benefit. Each employee will receive 1,000 non-dividend paying shares in the company in 1 year's time, provided the share price has increased from its current level of \$1.00 to at least \$1.50.

You may assume the following parameters:

- Risk-free interest rate: 4% per year continuously compounded
- Stock price volatility: 30% per year.

- (i) (a) Explain, by considering the components of the Black–Scholes formula, how this formula can be used to value the benefit to each employee.
- (b) Calculate the value of the benefit to each employee.

[9]

The company now wishes to limit the gain to each employee to a maximum of \$2,000.

- (ii) (a) Set out how your answer to part (i)(a) may differ, if this limit is introduced.
- (b) Calculate the value to the employee of this revised benefit.

[6]

An employee believes the original benefit, without the upper limit, is worth \$300. They have determined this by assuming that a 30% chance of the share price being at least \$1.50 in a year's time, and applying this to the current share price.

- (iii) Comment on the employee's approach.

[3]

Now, assume that a market exists in which the benefit can be traded, by market-makers, as an investment contract.

- (iv) Discuss the implications, in such a market, arising from the employee's valuation of the benefit.

[3]

[Total 21]

- 3 An insurance company holds the following credit-risky assets with 2 years to maturity:

<i>Credit rating</i>	<i>Number of assets held at time <math>t = 0</math></i>	<i>Description</i>
A	1,000	Highest rating
B	1,000	Second highest rating
C	1,000	Third highest rating
D	0	Default

The transitions that can occur between credit ratings in any year are set out in the following table:

	<i>Year 1 (%)</i>	<i>Year 2 (%)</i>
Exactly one rating downgrade	20	15
Exactly two rating downgrade	10	8
Exactly three rating downgrade	4	2

Downgrades that would take an asset lower than credit rating D should be ignored.

There can be at most one transition to a different rating in a single year. There are no upgrades. There are no other ways that an asset can leave or enter the insurer's credit portfolio.

- (i) Calculate, for each year, the transition matrix that shows the expected number of assets moving between credit states. [8]
- (ii) Construct, for each year, the 1-year transition rate matrix using your answer from part (i). [6]
- (iii) Calculate, for each credit rating at time  $t = 0$ , the probability that a default would have occurred by time  $t = 2$ . [8]

A 2-year zero-coupon risk-free bond  $P(0,2)$  has the following features at time  $t = 0$ :

<i>Redemption</i>	<i>Maturity</i>
\$1,000	2

A credit-risky bond has the following features at time  $t = 0$ :

<i>Redemption</i>	<i>Maturity</i>	<i>Credit rating</i>	<i>Recovery rate on default</i>
\$1,000	2	A	75%

The risk-free force of interest is 5% p.a.

- (iv) Calculate the implied credit spread (i.e. the return above the risk-free rate) for the credit-risky bond. [7]
- (v) Explain, without carrying out any further calculations, how the credit spread may change if the probability of default increased. [3]

[Total 32]

- 4 An insurance company's cumulative incurred claims for the last 5 accident years are given in the following table:

<i>Accident year</i>	<i>Development year</i>				
	<i>0</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>
2016	143	165	173	183	190
2017	150	173	186	199	
2018	167	180	197		
2019	160	175			
2020	172				

It can be assumed that claims are fully run off after 4 years. The premiums received for each year are:

<i>Accident year</i>	<i>Premium</i>
2016	201
2017	210
2018	215
2019	222
2020	228

You do not need to make any allowance for inflation.

- (i) (a) Calculate the reserve at the end of 2020 using the basic chain ladder method.
- (b) Calculate the reserve at the end of 2020 using the Bornhuetter–Ferguson method.

[24]

- (ii) Comment on the differences in the reserves produced by the methods in part (i).

[3]

[Total 27]

**END OF PAPER**