

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

14 April 2023 (am)

Subject CM1 – Actuarial Mathematics Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the
Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on
T. 0044 (0) 1865 268 873.

- 1 (i) Describe in words ${}_{a|b}q_{[x]+1}$. [2]
- (ii) Calculate, showing all working, ${}_{5|3}q_{[38]+1}$.

Basis:

Mortality AM92 (select) [2]
 [Total 4]

- 2 A general insurance company has set up a generalised cashflow model for the **claims** payments it will pay arising from its portfolio of car insurance policies.

Describe the cashflows with reference to the certainty and uncertainty of size and timings of payments from the insurance company's point of view. [3]

- 3 Calculate $a_{82\frac{1}{4}}^{(4)}$, showing all working.

Basis:

Mortality PMA92C20
 Interest 4% per annum effective

[7]

- 4 A life insurance company sells 1-year pet insurance policies. If the pet dies during the year, the sum assured is payable at the end of the year. If the pet escapes, no benefit is payable and the policy is cancelled.

The company uses the following dependent decrement rates to price these policies:

$(aq)^{death}$	0.25
$(aq)^{escape}$	0.35

The company assumes that the forces of decrement are constant over the year, that the two decrements operate independently and that the independent and dependent forces of decrement are equal.

The company has proposed reducing the underlying **independent** mortality rate to 70% of the current value.

Calculate, showing all working, the proposed new dependent decrement rates. [7]

- 5 The gross future loss random variable at inception for a policy issued to a life, aged 50 exact, is given by:

$$1,500a_{\overline{\min(K_{50}, 15)}|} + 35\left(\ddot{a}_{\overline{\min(K_{50}+1, 15)}|} - 1\right) + 200 - 0.97 \times P$$

where:

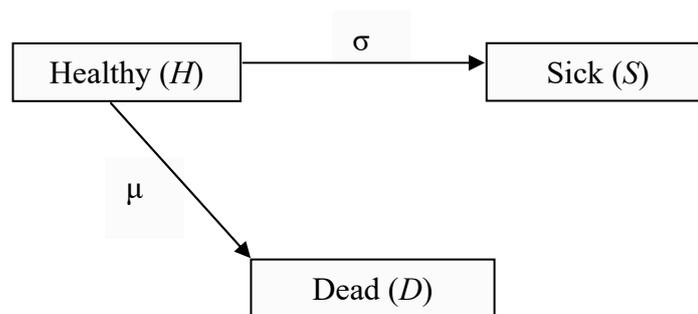
P = premium paid.

K_x = curtate future lifetime of a person aged x exact.

Describe in words the timings and amounts of the benefits, expenses and premium of the policy implied by the gross future loss random variable. [5]

- 6 A life insurance company sells a special 15-year pure endowment assurance policy, where a sum assured of \$10,000 is paid at the end of the term if the policyholder remains healthy throughout.

The following three-state transition model is used:



- (i) Show that the expected present value of the benefit, in respect of a healthy life aged 50 exact, is approximately equal to \$1,054. [4]

Premiums of P per annum are payable continuously throughout the policy term, ceasing on death or if the policyholder becomes sick.

- (ii) Calculate P , showing all working. [5]

Basis:

$$\mu = 0.04$$

$$\sigma = 0.08$$

Force of interest: 3% per annum

[Total 9]

7 An organisation is required to pay \$5 million in 18 years' time. It holds a cash fund equal to the present value of the liabilities and wishes to use the whole fund to purchase a combination of 9-year and 20-year zero-coupon bonds in order to immunise against small changes in the interest rate. The current interest rate is 8% per annum effective.

- (i) Calculate, showing all working, the proportion of the cash fund that should be invested in each of the two zero-coupon bonds. [8]

The organisation invests in the two bonds in accordance with the proportions calculated in part (i).

- (ii) Estimate, using the volatility, the percentage change in the present value of the assets, if the interest rate were to increase to 8.5% per annum effective. You should show your working. [4]

[Total 12]

8 A life insurance company issues a retirement policy to a male policyholder aged 60 exact. The policy provides the following benefits:

- An immediate whole life level annuity of £25,000 per annum payable monthly in advance.
- A reversionary annuity of £15,000 per annum payable monthly in advance to the policyholder's surviving female spouse, who is currently aged 62 exact.
- A lump sum of £200,000 payable immediately to the policyholder's family if the policyholder dies before age 65.

Calculate the expected present value of the policy benefits at outset. You should show all working.

Basis:

Mortality	Male PMA92C20
	Female PFA92C20
Interest	4% per annum effective

[13]

- 9** A government issues a fixed interest bond paying coupons at a rate of 9% per annum, payable half-yearly in arrears. The bond is to be redeemed at \$110 per \$100 nominal on any coupon payment date from 10 to 15 years after issue. The date of redemption is at the discretion of the government.

Investor A is subject to income tax at 25% and capital gains tax at 30%, and wishes to achieve a net redemption yield of at least 6% per annum effective.

- (i) Calculate, showing all working, the maximum price per \$100 nominal that Investor A should offer for this bond on issue. [5]

Investor A purchases the bond at the price determined in part (i).

Three years after issue, immediately after a coupon payment has been made, Investor A decides to sell the bond to Investor B.

Investor B is subject to income tax at 10% and capital gains tax at 35%, and wishes to obtain a net redemption yield of at least 8% per annum effective.

- (ii) Calculate, showing all working, the maximum price per \$100 nominal that Investor A can expect to receive from Investor B. [7]

The bond is sold to Investor B at the price determined in part (ii).

- (iii) Calculate, using linear interpolation, the net effective annual redemption yield that will be obtained by Investor A. You must show all your working. [5]
[Total 17]

10 A life insurance company issued a 15-year special endowment assurance product on 1 January 2013, to a portfolio of policyholders aged 50 exact. The benefits were as follows:

- A death benefit, payable at the end of the year of death, of \$100,000 during the first policy year, which reduces by \$5,000 each subsequent year.
- Survival benefits of \$20,000 payable on survival to each of the 10th and 15th policy anniversaries.

Level annual premiums are payable, annually in advance, ceasing after 10 years, or on earlier death.

- (i) Show that the annual premium is approximately equal to \$2,786. You must show all your working. [11]

On 1 January 2022, there were 225 policies in force, and 2 policyholders died during 2022.

- (ii) Calculate, showing all working, the mortality profit for this portfolio for 2022. [10]

- (iii) Explain your results in part (ii). [2]

Basis:

Mortality AM92 Ultimate
Interest 6% per annum effective

[Total 23]

END OF PAPER