

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

29 September 2020 (am)

Subject SP6 – Financial Derivatives Specialist Principles

Time allowed: Three hours and fifteen minutes

<p>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</p>

If you encounter any issues during the examination please contact the Examination Team on T. 0044 (0) 1865 268 873.

- 1** An insurance company has a portfolio of products with a variety of guaranteed surrender values and annuity options. These liabilities are managed through the use of duration hedging and derivatives, including swaptions.
- (i) Describe the payoffs from European payer and receiver swaptions. [2]
 - (ii) Derive a put–call parity relationship for European swaptions. [4]
 - (iii) Suggest why the extensive use of derivatives could increase liquidity risk for the insurer. [2]
 - (iv) Propose how the insurer could monitor its liquidity risk relating to the use of derivatives. [3]
- [Total 11]

- 2** A chooser option is an option where, after a specified period of time, the holder can choose whether the option is a call or a put.

A chooser option is written on an underlying asset of a stock that does not pay dividends, with a price S_t at time t . The time when the choice is made is in T_1 years from the current time. The underlying option is assumed to be vanilla, with strike price K and European, with a time to maturity in T_2 years ($T_2 > T_1$) from the current time. Let r be the risk-free interest rate and c the value of the underlying call option in time T_1 years.

- (i) Demonstrate that the value of this chooser option in T_1 years is

$$c + \max \left\{ 0, Ke^{-r(T_2-T_1)} - S_{T_1} \right\}. \quad [4]$$

- (ii) Explain how the equation in part (i), when viewed in terms of call and put options, can be used to price the chooser option at the current time. [4]

A company has announced that it is bidding for a large government contract and will find out whether or not it has been successful in 2 months' time.

- (iii) Suggest how a chooser option could be useful to a shareholder in the company. [4]
- [Total 12]

- 3** The trustee of a pension fund and the sponsoring employer have agreed to target a full transfer of the fund's liabilities to an insurance company within the next 3 years. The cost of the transfer will be met in full by the sponsoring employer.

To minimise the risk to the sponsoring employer, the trustee has agreed to immediately transition to a very low-risk investment strategy.

The fund currently includes a £200 million holding in a diversified UK property fund that has a 2-year disinvestment period.

- (i) Describe property swaps. [2]
- (ii) Explain how the trustee could use property swaps to mitigate the exposure to the property fund. [2]
- (iii) Describe which risks are mitigated and which risks are introduced by entering into a property swap. [2]
- (iv) Compare how the trustee could use index-linked bonds, inflation swaps or Limited Price Indexation (LPI) swaps to mitigate the exposure to inflation inherent in the pension fund's liabilities. [5]

When discussing this task with one of your colleagues, they suggest that you consider using overseas inflation-linked bonds to hedge the inflation risk.

- (v) Evaluate this proposal. [4]
- [Total 15]

4 An insurance company is designing a new product to sell to customers who are home owners with no remaining mortgage. Customers are able to borrow a fixed loan amount (L) from the insurer at time 0. The maximum amount they can borrow is 60% of the market value of their house (H) at time 0.

- (i) Give the cashflows at time 0 between the customer and the insurance company. [1]

The loan has a fixed interest rate of r p.a., continuously compounded. There are no further cashflows between the customer and insurance company until the customer dies. When the customer dies the house is sold by the customer's estate and the money is used to repay the loan in full. If there is any remaining money after repaying the insurance company then it is retained in the customer's estate. If there is insufficient money to repay the loan then the customer's estate is required to find additional funds for the full repayment.

- (ii) Give the cashflows at the time of death between the customer's estate and the insurance company. [2]

In order to make this product more marketable the insurance company adds in a guarantee that the maximum amount repayable at the time of the death is the market value of the house at that time.

- (iii) Demonstrate that, by including the guarantee at the time of death (T), the amount of money received by the insurance company can be written as

$$\min\{H(T), L(T)\},$$

where $H(T)$ is the market price of the house at time T and $L(T)$ is the total loan amount outstanding at time T . [2]

- (iv) Express the guarantee as an option. [2]

The insurance company is considering using the Black–Scholes methodology to value this guarantee to determine pricing and reserving requirements.

- (v) Assess the suitability of using the Black–Scholes methodology to value this guarantee on the product relating to a single house. [7]

- (vi) Suggest ways in which the insurance company could hedge a portfolio consisting of the guarantee elements of the product. [4]

[Total 18]

5 The Black model is widely used to value European-style interest rate options.

- (i) Explain how Black's model can also be used to value European options on the future value of a currency. [2]
- (ii) State the formula for valuing a European-style put option on a bond under the Black model, defining all terms. [2]

A 3.25-year bond has the following details:

- The bond pays semi-annual coupons of €3, the next of which is due in 3 months.
 - Continuously compounded interest rates are 3.5% p.a.
 - The volatility of the forward bond price is 12%.
- (iii) Verify, using Black's model, that the value of an 18-month European put option on the 3.25-year bond, with strike price of €110 is €8.28. [6]
- (iv) Determine whether the option value would be more or less than €8.28 under each of the following three scenarios (keeping everything else constant):
- (a) The strike price is €95.
- (b) The semi-annual coupon rate is €4.
- (c) Continuously compounded interest rates are 5% p.a.

[3]

[Total 13]

6 Let C be the current price of a call option written on an underlying with price S_t at time t , strike price K , risk free rate r and maturity time T . Let $f(x)$ be the risk neutral probability density function.

- (i) Describe the finite difference method for valuing derivatives. [1]
- (ii) Describe the implicit finite difference method. [7]
- (iii) Demonstrate that the current price of the call option is

$$C = \int_K^{\infty} e^{-rT} (S_T - K) f(S_T) dS_T . \quad [3]$$

There is a deep market in call options written on the underlying asset with a wide range of strike prices and maturity dates available.

- (iv) Give two reasons why a call option price changes if the strike price is increased by a small positive amount dK . [2]
- (v) Demonstrate that $F(K) = e^{rT} \frac{\partial C}{\partial K} + 1$, where F is the risk-neutral cumulative distribution function. [3]

[Total 16]

7 A European airline is reviewing its fuel hedging policy. It has historically used heating oil futures contracts and US Dollar (USD)/Euro (EUR) currency forwards to hedge this risk.

(i) Describe the differences between futures and forward contracts. [3]

A proposal is being made to the management board to increase fuel hedging and currency hedging to 100% of expected demand over the next 12 months using heating oil futures and Euro (USD/EUR) currency futures.

You are given the following company and derivative market information:

Company information

<i>Quarter</i>	<i>Expected demand (millions of gallons)</i>
Q1	20
Q2	32
Q3	36
Q4	28

Contract specifications

<i>Contract</i>	<i>Heating oil</i>	<i>Euro futures</i>
Price quotation	Dollars and cents per gallon	USD per EUR
Contract unit	42,000 gallons	€125,000
Contract months	Quarterly; March, June, September, December	Quarterly; March, June, September, December
Minimum price fluctuation	\$0.0001 per gallon (\$4.20 per contract)	\$0.00005 per € (\$6.25 per contract)
Settlement method	Deliverable	Physical delivery

Market information

<i>Heating oil</i>	<i>Price</i>
Mar	1.67
Jun	1.58
Sept	1.52
Dec	1.50

<i>Euro futures</i>	<i>Price</i>
Mar	1.11
Jun	1.13
Sept	1.16
Dec	1.20

- (ii) Calculate the number of heating oil and Euro future contracts that the airline would need to buy or sell to hedge 100% of expected demand over the first 6 months of the next year. Assume that March contracts would be used to hedge its Q1 exposure and June contracts for Q2 exposure. [5]

Having reviewed the proposal, the board has queried the use of longer-dated contracts to hedge the expected exposure from Q3 and Q4, due to the lower levels of liquidity in these contracts.

- (iii) Recommend, with reasons, an alternative trading strategy that could be used to address this concern. [3]

The airline's main competitor has chosen not to hedge any of its fuel costs.

- (iv) Discuss why the competitor may have chosen this strategy. [4]
[Total 15]

END OF PAPER