

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2022

CM1 - Actuarial Mathematics Core Principles Paper A

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the Specialist Advanced (SA) and Specialist Principles (SP) subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
July 2022

A. General comments on the *aims of this subject and how it is marked*

CM1 provides a grounding in the principles of modelling as applied to actuarial work - focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations, but candidates are not penalised for this. However, candidates may not be awarded full marks where excessive rounding has been used or where insufficient working is shown.

Although the solutions show full actuarial notation, candidates were generally expected to use standard keystrokes in their solutions.

Candidates should pay attention to any instructions included in questions. Failure to do so can lead to fewer marks being awarded. In particular, where the instruction, “*showing all working*” is included and the candidate shows little or no working, then the candidate will be awarded very few marks even if the final answer is correct.

Where a question specifies a method to use (e.g. *determine the present value of income using annuity functions*) then where a candidate uses a different method the candidate will not be awarded full marks.

Candidates are advised to familiarise themselves with the meaning of the command verbs (e.g. state, determine, calculate). These identify what needs to be included in answers in order to be awarded full marks.

B. Comments on *candidate performance in this diet of the examination.*

The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those scripts.

Many candidates appeared to be inadequately prepared, in terms of not having sufficiently covered the entire breadth of the subject. We would advise candidates not to underestimate the quantity of study required for this subject.

Candidates should be aware that the questions cannot be answered using knowledge alone and well prepared candidates will demonstrate application of their knowledge to the questions asked.

Where candidates made numerical errors, the examiners awarded marks for the correct method used and for the parts of the calculation that were correct. However, many candidates often did not show enough of their working to fully benefit from this.

The Examiners felt that the “open book” nature of the online exam led some candidates to rely on their notes much more than if the exam had been “closed book”. The Examiners strongly recommend that candidates prepare for online exams just as thoroughly as they would do if the exam were of the traditional “closed book” format. Candidates are recommended to use their notes only as a tool to check or confirm answers where necessary, rather than as a source for looking up the answers.

C. Pass Mark

The Pass Mark for this exam was 58.
1644 presented themselves and 520 passed.

Solutions for Subject CM1 Paper A- April 2022

Q1

Number of shares held	[½]
Historical dividend amounts, current dividend (or current price of share and dividend yield)	[½]
Historical Dividend dates	[½]
Ex-dividend date of equity share	[½]
Retained earnings	[½]
Growth prospect of the company, sector or economy in general	[½]
Other valid answers accepted	[½]

[Marks available 3½, maximum 2]

This question was generally well answered. Full marks were not awarded where candidates included data items that did not relate to the holding of the equity share in a single company, and/or which were not relevant to modelling the future value of the equity shareholding.

Q2

Price (per £100 nominal) of the 6-month investment is given by:

$$P = 100 \times \left(1 - \frac{6}{12} \times 0.0415 \right) = 97.925$$

[1½]

Thus, equivalent effective rate of interest per annum, i , is given by:

$$97.925 \times (1+i)^{\frac{6}{12}} = 100 \Rightarrow (1+i)^{\frac{6}{12}} = 1.0212 \Rightarrow i = 0.04283 \text{ or } 4.283\% \text{ per annum}$$

[1½]

Since 4.35% > 4.283% the Bank Deposit offers a better return.

[1]

[Total 4]

A common error made by candidates, was to treat the simple discount rate as a compound discount factor.

Q3

The present value of the liabilities is

$$V_L = 4v^7 + 13v^{11} = 9.508466 \quad [1]$$

and the present value of the assets is

$$V_A = 6.9617 * v^4 + 11.4007 * v^{18} = 9.508483 \quad [1]$$

These are the same (to 4 decimal places) and so Redington's first condition is satisfied, present values equal [1/2]

The second condition is that the volatility of the assets and liabilities should be equal:

$$-V_L' = 28v^8 + 143v^{12} = 88.634165 \quad [1]$$

$$\frac{-V_L'}{V_L} = \frac{88.6341}{9.5084} = 9.3216 \quad [1/2]$$

$$-V_A' = 4 * 6.9617 * v^5 + 18 * 11.4007 * v^{19} = 88.634183 \quad [1]$$

$$\frac{-V_A'}{V_A} = \frac{88.6341}{9.5084} = 9.3216 \quad [1/2]$$

and since they are the same (to 4 decimal places) the second condition of volatility equal is also satisfied. [1/2]

[Total 6]

Acceptable alternatives for the second condition (having shown the first condition holds) was showing the equality of the Discounted Mean Terms of the assets and liabilities, the numerator of the DMT or the numerator of the Volatility, if accompanied by the reasoning that as the denominators are the same, only the numerators needed to be considered.

Some candidates did not calculate the required values for assets and liabilities but assumed the values were the same. Some candidates did not present their answers to sufficient decimal places, so full marks were not awarded when calculations were not shown. These answers were penalized as the candidate had not sufficiently demonstrated that the values were equal. The easiest way to show full workings is to include the values of the discount factors as shown in the solution above.

Q4

(a)

$$\text{Variance} = \frac{1}{d^2} \left[{}^2A_{\overline{x:n+1}|} - (A_{\overline{x:n+1}|})^2 \right] \text{ where } {}^2A_{\overline{x:n+1}|} \text{ is } A_{\overline{x:n+1}|} \text{ calculated at } i' = (1+i)^2 - 1 \quad [2\frac{1}{2}]$$

(b)

$$\begin{aligned} \text{Variance} &= 5,000^2 \frac{1}{d^2} \left[{}^2A_{44:\overline{21}|} - (A_{44:\overline{21}|})^2 \right] \\ &= \frac{5,000^2}{d^2} \left[{}^2A_{44} - v^{2 \times 21} \frac{l_{65}}{l_{44}} {}^2A_{65} + v^{2 \times 21} \frac{l_{65}}{l_{44}} - (A_{44:\overline{21}|})^2 \right] \end{aligned}$$

$$\begin{aligned}
 &= \frac{5,000^2}{0.00147929} \left[\begin{array}{l} 0.08856 - 0.192575 \times \left(\frac{8,821.2612}{9,814.3359} \right) \times 0.30855 + 0.192575 \times \left(\frac{8,821.2612}{9,814.3359} \right) \\ -(0.45258)^2 \end{array} \right] \\
 &= \frac{5,000^2}{0.00147929} [0.03515 + 0.17309 - 0.20483] \\
 &= \text{£}^2 57.69m
 \end{aligned}$$

[4½]
[Total 7]

Common errors included: -

In (a) failing to write down the new interest rate, referring to 'n' rather than 'n+1'.

In (b) omitting the 5,000², being unable to calculate the ${}^2A_{44:\overline{2}|}$ factor.

Q5

Let μ_x^k be the independent force of decrement k in year x where $k = d, w, t$ for deaths, withdrawals, and transfers respectively.

The probability that a student is active at the end of year 1, before any student qualifies:

$$= \exp [-(\mu_1^d + \mu_1^w + \mu_1^t)] = e^{-0.45} = 0.63763 \quad [1]$$

The probability of a student at the company qualifying at the end of year 1:

$$= 0.63763 \times 0.03 = 0.0191289 \quad [1]$$

The probability that a student is active at the end of year 2, given they were active at the start of year 2, and before any student qualifies:

$$= \exp [-(\mu_2^d + \mu_2^w + \mu_2^t)] = e^{-0.475} = 0.62189 \quad [½]$$

The probability that a student is still at the company and not yet qualified at the end of year 2:

$$0.63763 \times 0.62189 \times 0.97 = 0.384635477 \quad [1]$$

The probability of a student at the company qualifying at the end of year 2 is:

$$0.384635477 \times 0.12 = 0.04616 \quad [½]$$

The probability that a student is active at the end of year 3, given they were active at the start of year 3, and before any student qualifies:

$$= \exp [-(\mu_3^d + \mu_3^w + \mu_3^t)] = e^{-0.48} = 0.61878 \quad [½]$$

The probability that a student is still at the company and not yet qualified at the end of year 3:

$$0.384635477 \times 0.88 \times 0.61878 = 0.20945 \quad [1]$$

The probability of a student at the company qualifying at the end of year 3 is:

$$0.20945 \times 1 = 0.20945 \quad [½]$$

Thus, the total probability of qualifying whilst still with the company:

$$= 0.01913 + 0.04616 + 0.20945 = 0.27474 \quad [1]$$

[Total 7]

A common error made by candidates when calculating the probability that a student is still at the company and not yet qualified at the end of years 2 and 3 was ignoring the students needing to survive the previous years' death, withdrawal, transfer and qualifying decrements.

Q6

- (a) A lump sum of £50,000 is payable immediately on death whether from the healthy state or from the sick state. The period of cover is to age 55 (for 30 years).
- (b) A lump sum payable immediately on entering the sick state (for the first and any subsequent bouts of sickness). Where the amount payable is £20,000 multiplied by the probability that the policyholder will be continuously sick for at least six months and then die from the sick state before age 55. The cover is to age 54.5 (for 29.5 years).
- (c) £5,000 per annum payable continuously whilst a policyholder is sick (for the first and any subsequent bouts of sickness). The period of cover is to age 55 (for 30 years).

[Total 6]

Although the benefit described in part (b) could exist and could be described fully, it was not the benefit the examiners had intended to present and neither was it one which candidates were used to seeing. Adding an extra term $e^{-\delta(s+0.5)}$ to the second integral the solution becomes: -

A lump sum of £20,000 is payable immediately on death from the sick state having been continuously sick for at least six months at the time of death. The cover is from age 25.5 to 55 (for 29.5 years).

The question as it appears in the paper is harder than the examiners had intended. We recognised this and to mitigate the increased difficulty, marks were awarded either for the correct solution as it appeared in the paper or for the solution to the intended question.

Q7

We have $i^{(2)} = 0.049390$ and $\frac{D}{R} \times (1 - t_1) = \frac{6}{103} \times (1 - 0.20) = 0.046602$

$0.049390 > 0.046602$

Since $i^{(2)} > \frac{D}{R}(1 - t_1)$ there is a capital gain on redemption and the worst-case scenario for the investor is if the stock is redeemed as late as possible (i.e., at time 15).

The price (per £100 nominal) is:

$$\begin{aligned} P &= 6 \times (1 - 0.20) \times a_{\overline{15}|}^{(2)} + 103v^{15} - 0.25 \times (103 - P)v^{15} \\ \Rightarrow (1 - 0.25 \times 0.48102) \times P &= 4.8 \times 1.012348 \times 10.3797 + 0.75 \times 103 \times 0.48102 \\ \Rightarrow P &= \text{£}99.57 \end{aligned}$$

[Total 6]

This question was generally well answered.

Q8

- (i)

$$\bar{A}_{xy:\overline{n}|}^2 = \int_0^n e^{-\delta t} {}_t p_x (1 - {}_t p_y) \mu_{x+t} dt \quad [1]$$

$$= \int_0^n e^{-\delta t} {}_t p_x \mu_{x+t} dt - \int_0^n e^{-\delta t} {}_t p_{xy} \mu_{x+t} dt = \bar{A}_{x:\overline{n}|}^1 - \bar{A}_{xy:\overline{n}|}^1 \quad [1]$$

(ii)

Using this result with $x = y = 50$ and $n = 1$

$$\bar{A}_{50:\overline{1}|}^1 - \bar{A}_{50:50:\overline{1}|}^1 = \bar{A}_{50:\overline{1}|}^1 - \frac{1}{2} \cdot \bar{A}_{\overline{1}|}^1 \quad [1]$$

$$\bar{A}_{50:\overline{1}|}^1 = (1.07)^{\frac{1}{2}} \cdot q_{50} v$$

$$= 0.002508 \times (1.07)^{-\frac{1}{2}} = 0.00242458 \quad [1\frac{1}{2}]$$

$$\bar{A}_{\overline{1}|}^1 = (1.07)^{\frac{1}{2}} \cdot v \cdot (1 - p_{50}^2)$$

$$\bar{A}_{50:50:\overline{1}|}^1 = (1.07)^{-\frac{1}{2}} (1 - (1 - 0.002508)^2) = 0.00484307 \quad [1\frac{1}{2}]$$

$$100,000 \bar{A}_{xy:\overline{n}|}^2 = 100,000 \times (0.00242458 - \frac{1}{2} \times 0.00484307)$$

$$= 0.3040$$

[1]

[Total 7]

A common error made by candidates in part (i) was to just write out the benefits in integral form, but not show how the equality was derived.

In part (ii) candidates who correctly answered the question using an integral approach or otherwise were given credit.

Q9

(i) $f_{3,1}$ is such that:

$$1.053 \times (1 + f_{3,1}) = 1.059^2 \quad [1]$$

$$\Rightarrow f_{3,1} = 6.5034\% \text{ pa} \quad [1]$$

(ii) Let $i_y = y$ -year spot rate p.a.

$$i_1 = f_{0,1} = 4.2000\% \quad [1]$$

$$(1 + i_2)^2 = 1.042 \times 1.048$$

$$\Rightarrow i_2 = 4.4996\%$$

$$(1 + i_3)^3 = 1.042 \times 1.048 \times 1.053 \quad [1]$$

$$\Rightarrow i_3 = 4.7657\% \quad [1]$$

$$(1+i_4)^4 = 1.042 \times 1.048 \times 1.053 \times 1.065034$$

$$\Rightarrow i_4 = 5.1974\% \quad [1]$$

(iii)

PV of payments of bond is:

$$P = 2.5(1.042^{-1} + 1.044996^{-2} + 1.047657^{-3} + 1.051974^{-4}) + 105 \times 1.051974^{-4}$$

$$\Rightarrow P = 94.6412 \quad [3]$$

The equation of value to find the gross redemption yield is:

$$94.6412 = 2.5 a_{\overline{4}|i} + 105 v^4 \quad [1]$$

Try 5%, RHS = $2.5 \times 3.5460 + 105 \times 0.82270$
 = 95.2485 [½]

Try 6%, RHS = $2.5 \times 3.4651 + 105 \times 0.79209$
 = 91.8322 [½]

$$\Rightarrow i = 0.05 + \frac{95.2485 - 94.6412}{95.2485 - 91.8322} \times 0.01 \quad [½]$$

$$= 0.05178 \quad (\text{i.e. } 5.178\% \text{ pa}) \quad (\text{accurate answer } 5.1746\%) \quad [½]$$

[Total 12]

Parts (i) and (ii) were generally well answered. Candidates who did not present their answer as a percentage to four decimal places did not receive full marks.

In part (iii) many candidates missed out the derivation of the price of the bond using the term structure of interest rates given in the question. Candidates also often incorrectly used coupon payments of 0.025 or 0.25 instead of 2.5.

Where the full workings for the linear interpolation were not shown, candidates did not receive full marks.

Q10

(i)

$$(a) = (1 - 0.004251) \times 0.05 = 0.049787$$

$$(b) = 1 - .004251 - 0.049787 = 0.945962$$

$$(c) = 0.896805 \times 1 = 0.896805$$

$$(d) = 0.896805 \times 0.945962 = 0.848343 \quad [2]$$

(ii)

Interest earned on cashflows is given by

$$\left(\frac{242.75}{5,000 - 20 - 125} \right) = 5\% \quad [1]$$

Interest in year 1,

$$(e) = 0.05 \times (5,000 - 200 - 1,250) = 177.50 \quad [1/2]$$

Renewal expense in year 3 is the same as in year 2, therefore (f) = 20.

Renewal commission in year 3 is the same as in year 2, therefore (g) = 125.

Interest in year 3 is the same as in year 2, therefore (h) = 242.75. [1 1/2]

Sum Assured is given by

$$\frac{85.02}{0.004251} = 20,000 \quad [1/2]$$

Expected cost of death payment in year 1

$$(i) = 20,000 \times 0.00355 = 71 \quad [1/2]$$

Expected cost of surrender payment in year 2

$$(j) = (2 \times 5,000) \times 0.049787 = 497.87 \quad [1/2]$$

Expected cost of maturity payment in year 3

$$(k) = 20,000 \times 0.994927 = 19,898.54 \quad [1/2]$$

Alternative method $20,000 - 101.46 = 19,898.54$

Profit vector for year 1

$$(l) = 5,000 - 200 - 1,250 + 177.5 - 71 - 498.23 = 3,158.27 \quad [1/2]$$

Profit vector for year 3

$$(m) = 5,000 - 20 - 125 + 242.75 - 101.46 - 19,898.54 = -14,902.25 \quad [1/2]$$

Profit Signature for year 1

$$(n) = 3,158.27 \times 1 = 3,158.27 \quad [1/2]$$

Profit Signature for year 3

$$(o) = -14,902.25 \times 0.848343 = -12,642.22 \quad [1/2]$$

Risk discount rate is given by

$$\sqrt{\left(\frac{4,048.95}{3,407.92} \right)} - 1 = 9\% \quad [1/2]$$

Discount factor for year 1

$$(p) = (1.09)^{-1} = 0.917431$$

Discount factor for year 2

$$(q) = (1.09)^{-2} = 0.841680$$

Discount factor for year 3

$$(r) = (1.09)^{-3} = 0.772183 \quad [1\frac{1}{2}]$$

PV profit for year 1

$$(s) = 3,158.27 \times 0.917431 = 2,897.49$$

PV profit for year 3

$$(t) = -12,642.23 \times 0.772183 = -9,762.12$$

[1]

[Total 12]

This was a new type of profit testing question which required some logical thinking. Candidates did not appear to be intimidated by this new presentation and generally performed well.

A common error was not showing all workings.

Q11

Let P denote the monthly premium.

Then, equation of value is:

$$12P \times \ddot{a}_{[30]:\overline{35}|}^{(12)} = 250,000 \times \bar{A}_{[30]:\overline{35}|} + 0.57P + 0.03 \times 12P \times \ddot{a}_{[30]:\overline{35}|}^{(12)} + 250 \quad [3\frac{1}{2}]$$

From tables, we have:

$$\ddot{a}_{[30]:\overline{35}|}^{(12)} = \ddot{a}_{[30]:\overline{35}|} - \frac{11}{24} \times \left(1 - v^{35} \frac{l_{65}}{l_{30}} \right) = 19.072 - \frac{11}{24} \times \left(1 - 1.04^{-35} \frac{8821.2612}{9923.7497} \right) = 18.7169$$

$$\bar{A}_{[30]:\overline{35}|} = 1.04^{0.5} \times \left(A_{[30]} - v^{35} \frac{l_{65}}{l_{30}} A_{65} \right) = 1.04^{0.5} \times \left(0.16011 - 1.04^{-35} \frac{8821.2612}{9923.7497} 0.52786 \right) = 0.04202$$

[3]

Thus, we have:

$$\begin{aligned} 12P \times 18.7169 &= 250,000 \times 0.04202 + 0.57P + 0.36P \times 18.7169 + 250 \\ \Rightarrow 217.2947 \times P &= 10,755 \\ \Rightarrow P &= \$49.49 \end{aligned}$$

[½]

(ii)

Retrospective gross premium reserve at time 25 is given by:

$${}_{25}V = v^{-25} \frac{l_{[30]}}{l_{55}} \times \left(0.97 \times 12 \times 50 \times \ddot{a}_{[30]:25}^{(12)} - 0.57 \times 50 - 250,000 \times \bar{A}_{[30]:25}^1 - 250 \right) \text{ at } 6\% \text{ p.a.} \quad [2\frac{1}{2}]$$

Thus, at 6% p.a. interest, we have:

$$\begin{aligned} v_{6\%}^{25} \times \frac{l_{55}}{l_{[30]}} &= 0.23300 \times \frac{9,557.8179}{9,923.7497} = 0.224408 \\ \ddot{a}_{[30]:25}^{(12)} &= \ddot{a}_{[30]}^{(12)} - v_{6\%}^{25} \times \frac{l_{55}}{l_{[30]}} \times \ddot{a}_{55}^{(12)} = \left(16.374 - \frac{11}{24} \right) - 0.224408 \times \left(13.057 - \frac{11}{24} \right) = 13.0884 \\ \bar{A}_{[30]:25}^1 &= 1.06^{0.5} \times \left(A_{[30]} - v_{6\%}^{25} \times \frac{l_{55}}{l_{[30]}} \times A_{55} \right) = 1.06^{0.5} \times (0.07316 - 0.224408 \times 0.26092) = 0.015039 \end{aligned} \quad [3]$$

Thus, the reserve at time 25 is given by:

$${}_{25}V = \frac{1}{0.224408} \times (0.97 \times 12 \times 50 \times 13.0884 - 0.57 \times 50 - 250,000 \times 0.015039 - 250) = \$15,950 \quad [1\frac{1}{2}]$$

(iii)

Given that the premium of \$50 is near to the \$49.49 calculated in part (i) we can reasonably assume that the basis too may be near to that given in part (i).

Using the premium basis with a rate of interest of 4% p.a., we have

$$GPV_{4\%}^{pro} = GPV_{4\%}^{ret} \quad [1]$$

Then, using a rate of interest of 6% p.a. rather than 4% p.a., the retrospective provision at time 25 will increase (as PV of past income will be accumulated at a higher rate of interest).

$$\text{Thus, we have } GPV_{6\%}^{ret} > GPV_{4\%}^{ret} \quad [1]$$

However, using a rate of interest of 6% p.a. rather than 4% p.a., the prospective provision at time 25 will decrease (as PV of future outgo will be discounted at a higher rate of interest).

$$\text{Thus, we have } GPV_{6\%}^{pro} < GPV_{4\%}^{pro} \quad [1]$$

$$\text{Hence, we have } GPV_{6\%}^{ret} > GPV_{4\%}^{ret} = GPV_{4\%}^{pro} > GPV_{6\%}^{pro} \quad [1\frac{1}{2}]$$

So the answer to (ii) would be lower if we calculated gross premium prospective reserves using the same basis. [1\frac{1}{2}]

[Total 17]

Part (i) Common errors included: -

Using an incorrect method for adjusting an annual annuity to monthly;

Allowing for the renewal expense incorrectly;

Not allowing for the death benefit to be payable immediately on death;

Using ultimate mortality instead of select mortality as specified in the basis.

In part (ii) candidates seem to be unfamiliar with the concept of retrospective reserves. Many candidates did not attempt this question part or calculated a prospective reserve, which gained very little credit.

A common error was not including the initial expenses when calculating the retrospective reserve.

In Part (iii) candidates performed very well. General comments about reserves gained very little credit.

Q12

(i)

$$50,000 + 50,000 \times 0.021 \times 10 = \text{£}60,500 \quad [1]$$

$$50,000 \times (1+k)^{10} = 60,500$$

$$(1+k)^{10} = 1.21 \Rightarrow k = 1.9244876\% \quad [1]$$

(ii)

Policy A

$$P\ddot{a}_{55:\overline{10}|} = 48,950A_{55:\overline{10}|}^1 + 1,050(IA)_{55:\overline{10}|}^1 + 60,500 \frac{l_{65}^{10}}{l_{55}} v_{6\%}^{10} \quad [3]$$

$$\frac{l_{65}^{10}}{l_{55}} v_{6\%}^{10} = \frac{8,821.2612}{9,557.8179} \times 0.558395 = 0.51536305 \quad [1/2]$$

$$(IA)_{55:\overline{10}|}^1 = (IA)_{55} - \frac{l_{65}^{10}}{l_{55}} \left[(IA)_{65} + 10A_{65} \right] \quad [1]$$

$$= 5.22868 - 0.51536305 \times [5.50985 + 10 \times 0.40177] = 0.318532773 \quad [1]$$

$$A_{55:\overline{10}|}^1 = A_{55} - \frac{l_{65}^{10}}{l_{55}} A_{65} = 0.26092 - 0.51536305 \times 0.40177 = 0.053862587 \quad [1]$$

$$\ddot{a}_{55:\overline{10}|} = 7.610 \quad [1/2]$$

$$48,950 \times 0.053862587 + 1,050 \times 0.318532773 + 60,500 \times 0.51536305$$

$$= \text{£}34,150.50$$

$$P \times 7.61 = 34,150.50 \Rightarrow P = \text{£}4,487.58 \quad [1/2]$$

Policy B

$$P\ddot{a}_{55:\overline{10}|} = \text{£}50,000 \left(q_{55} \times v^1 + (1.01924) \times {}_1|q_{55} \times v^2 + (1.01924)^2 \times {}_2|q_{55} \times v^3 \right. \\ \left. + \dots + (1.01924)^9 \times {}_9|q_{55} \times v^{10} \right)$$

$$+ \text{£}60,500 \frac{l_{65}^{10}}{l_{55}} v_{6\%}^{10}$$

The new interest rate is given by $\left(\frac{1.01924}{1.06} \right)^{-1} - 1 = 4\% \quad [1/2]$

$$P\ddot{a}_{55:\overline{10}|=6\%} = \frac{50,000}{1.01924} A_{55:\overline{10}|=4\%}^1 + 60,500 \frac{l_{65}^{10}}{l_{55}} v_{6\%}^{10} \quad [2 1/2]$$

$$A_{55:\overline{10}|i=4\%}^1 = A_{55} - \frac{l_{65} \times v_{4\%}^{10}}{l_{55}} A_{65} = 0.3895 - 0.623502985 \times 0.52786 = 0.060377714$$

[1]

$$P \times 7.61 = \frac{50,000}{1.0192} \times 0.060377714 + 60,500 \times 0.51536305 = 34,141.47951$$

[½]

$$\Rightarrow P = \text{£}4,486.40$$

[Total 14]

This question focused on the difference between simple and compound increases in benefits. Part (i) was generally well answered. In Part (ii) common errors for product A included: -

Treating benefit increases as compound increases rather than simple;

Not splitting the benefit into component parts (term assurances and a pure endowment);

Incorrectly calculating the benefit amounts for each of component parts of the benefit (level term assurance, increasing term assurance and pure endowment);

Using an incorrect formula for evaluating the increasing term assurance benefit.

In Part (ii) a common error for product B was incorrectly calculating the benefit amounts for each of the component of the benefits (term assurance and pure endowment).

[Paper Total 100]

END OF EXAMINERS' REPORT