

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

22 September 2022 (am)

Subject CM2 – Financial Engineering and Loss Reserving Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 An individual has the following utility function:

$$U(w) = \frac{(w^\gamma - 1)}{\gamma}, (w > 0),$$

where w is wealth in \$000s. Their current wealth is \$8,000 and their current utility is 2.1012.

- (i) Show that $\gamma = 0.01$ to two decimal places. [1]
- (ii) Show that $U(w)$ exhibits declining absolute risk aversion and constant relative risk aversion. [3]

The individual has been offered a ticket to enter a lottery with a 1 in 10,000 chance to win \$1m.

- (iii) Calculate, to the nearest \$, the maximum price, P , that the individual would pay for the ticket. [3]
 - (iv) Discuss why this form of utility function with $\gamma > 1$ would be inconsistent with common utility theory. [2]
- [Total 9]

2 (i) Describe the differences between a structural credit risk model and a reduced form credit risk model. [4]

A firm issues a 15-year zero-coupon bond with a maturity value of \$100m. The current value of the firm's assets is \$150m and the estimated volatility of the firm's assets is 33%. The risk-free rate of interest is 1% p.a. continuously compounded.

- (ii) Calculate the credit spread on the debt, using the Merton model. [6]
- [Total 10]

3 The table below gives the following values for a market as at time 0:

<i>Time t</i>	$f(t-1, t)$ (%)	$P(0, t)$ (\$)	$R(0, t)$ (%)	$B(t)$ (\$)
0	–	–	–	100.00
1	0.9	99.10	0.9	(a)
2	(b)	96.46	1.8	103.67
3	3.3	(c)	2.3	107.14
4	3.1	90.48	(d)	110.52

- $f(t, T)$ is the continuously compounded forward rate p.a. applying between time t and T .
- $P(t, T)$ is the price at time t for a zero-coupon bond maturing at time T , with a nominal value of \$100.
- $R(t, T)$ is the continuously compounded spot rate of interest p.a. at time t for the period t to T .
- $B(t)$ is the value of a cash account at time t .

(i) Calculate the values of (a), (b), (c) and (d) in the table above. [4]

At time 0, an investor purchased \$500 nominal of zero-coupon bonds that mature at time 3 and \$1,000 nominal of zero-coupon bonds that mature at time 4. At time 2, interest rate expectations have changed as set out below.

<i>Time t</i>	$f(t-1, t)$ (%)
2	–
3	2.5
4	2.0

(ii) Calculate the profit or loss the investor would make if they sold all of their bonds at time 2. [4]

- (iii) (a) Explain the meaning of an inverted yield curve.
- (b) Explain why an inverted yield curve is unusual.
- (c) Suggest possible reasons why a yield curve may be inverted.

[5]

[Total 13]

- 4** A financial derivative is held for a 2-year period. An analyst assumes that the change in the value of the derivative per year, i , is the same for each year. They assume that i follows a Normal distribution with parameters $\mu = -1$ and $\sigma = 1$. Let X_2 denote the accumulated value of this amount, i.e.

$$X_2 = (1 + i)^2.$$

- (i) Show that the probability that $X_2 \leq k$ for any non-negative k is given by

$$\frac{2}{\sqrt{2\pi}} \int_0^{\sqrt{k}} e^{-\frac{1}{2}x^2} dx. \quad [3]$$

- (ii) Show, by using an appropriate substitution or otherwise, that the answer to part (i) is the same as the probability that $G \leq k$, where G is a gamma distributed variable with parameters $\alpha = \frac{1}{2}$, $\lambda = \frac{1}{2}$. You may use without proof the fact that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. [4]

- (iii) Calculate the mean and variance of X_2 . [2]

- (iv) Discuss the appropriateness of the analyst's modelling assumptions. [4]

[Total 13]

- 5** A non-dividend paying stock has a price at time $t = 0$ of \$8. In any unit of time $(t, t + 1)$, the price of the stock either increases by 25% or decreases by 20%, and \$1 held in cash at time t receives interest to become \$1.04 at time $t + 1$. The stock price after t time units is denoted by S_t .

A derivative contract is written on the stock with expiry date $t = 2$, which pays \$10 if and only if S_2 is not \$8 (and otherwise pays \$0).

- (i) Explain what is meant by a risk-neutral probability measure. [2]

- (ii) Calculate the up-step and down-step probabilities under the risk-neutral probability measure for this model. [1]

- (iii) Calculate the price (at $t = 0$) of the derivative contract. [4]

[Total 7]

6 Consider a European call option, C, and a European put option, P, both written on a non-dividend paying stock, S, with the same strike price and maturity.

- (i) Determine, for C and P:
- (a) the put–call parity relationship by constructing and comparing two portfolios.
 - (b) a relationship between the deltas.
 - (c) a relationship between the gammas.

[6]

Consider now a portfolio of cash: n units of P and 1 million units of S. The delta of P is -0.212 , and the gamma of P is 0.377 .

- (ii) Calculate the value of n that would give a portfolio a delta of zero. [2]

Two derivatives are now added to the portfolio: the call option C and a new derivative, D, which has a delta of 0.222 and a gamma of 0.111 .

- (iii) Calculate the number of derivatives C and D that would need to be added to the portfolio so that both the delta and gamma of the expanded portfolio are zero.

[5]

[Total 13]

7 The run-off triangle below shows the cumulative claims incurred on a portfolio of general insurance policies.

<i>Accident year</i>	<i>Development year</i>			
	<i>0</i>	<i>1</i>	<i>2</i>	<i>3</i>
2018	1355	1876	2140	2288
2019	1456	2007	2232	
2020	1412	1986		
2021	1347			

The claims inflation over the 12 months up to the middle of the given year is as follows:

<i>Year</i>	<i>Rate (%)</i>
2019	2.00
2020	1.80
2021	2.40

It is estimated that corresponding claims inflation rates for future years will be as follows:

<i>Year</i>	<i>Rate (%)</i>
2022	2.80
2023	2.60
2024	1.90

- (i) Calculate the outstanding claims, using the inflation-adjusted chain ladder method. [10]
- (ii) Explain how you could validate whether the method in part (i) is appropriate for modelling this portfolio. [2]

Following a review, the insurer has decided to reduce the number of staff working on claim settlement.

- (iii) Discuss, without performing further calculations, how you may adapt your calculations in part (i) to reflect this change. [2]

The law requires the insurer to hold a reserve higher than the expected future claims to allow for possible adverse experience. The required reserve is $1.75 \times$ the present value of expected claims. Claims are assumed to be paid halfway through each year.

- (iv) Calculate the required reserve using the following discount rates:
 - (a) 3% p.a.
 - (b) 4% p.a. [2]
- (v) Calculate the implied duration of the insurer's reserve value. [1]
- (vi) Suggest criteria that the insurer may use to determine an appropriate asset in which to invest the reserve. [4]

[Total 21]

- 8** Consider the following assets in a world where the capital asset pricing model holds. These are the only risky assets in the market.

<i>Asset</i>	<i>Expected return (% p.a.)</i>	<i>Total value of assets in market (\$m)</i>	<i>Beta</i>
Risky asset A	3.5	20	1.5
Risky asset B	2.2	30	0.2
Risky asset C	4.4	10	2.4

- (i) Calculate:
- (a) the risk-free rate of interest.
- (b) the expected return on the market portfolio. [4]

The standard deviation of the return on the market portfolio is 10%.

- (ii) Calculate the market price of risk. [1]

The risk-free rate of interest now increases to 3% p.a.

- (iii) Explain why one or more of the figures in the table must change. [2]
[Total 7]

- 9** A pension fund has been offered two investment opportunities.

Asset A gives an annual return of $3B\%$, where B is a binomial random variable with parameters $n = 4$ and $p = 0.4$.

Asset B gives an annual return of $4P\%$, where P is a Poisson random variable with parameter $\mu = 2$.

Calculate the following three measures of investment risk for each asset:

- (a) Variance [1]
- (b) Semi-variance [4]
- (c) Shortfall probability versus a benchmark return of 4%. [2]
[Total 7]

END OF PAPER