# INSTITUTE AND FACULTY OF ACTUARIES

# EXAMINERS' REPORT

April 2022

## SP6 - Financial Derivative Principles
## Specialist Principles

**Introduction**

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the Specialist Advanced (SA) and Specialist Principles (SP) subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
July 2022


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## A. General comments on the *aims of this subject and how it is marked*

The aim of Financial Derivatives Principles (SP6) is to develop a candidate's ability to understand different types of financial derivatives and their uses, the markets in which they are traded, methods of valuation of financial derivatives, and the assessment and management of risks associated with a portfolio of derivatives. It builds on material covered in earlier subjects, particularly Loss Reserving and Financial Engineering (CM2).

Candidates are reminded to ensure that their answers are sufficiently detailed to demonstrate their understanding, as well as to make sure that more obvious points are still made to gain full marks. The model solutions are intended to reflect the level of detail that a high scoring candidate might be able to produce. For many questions there are more marks available than the question requires to achieve full marks. This reflects that the examiners will give credit for valid alternative solutions, particularly in questions focussed on higher level skills.

Candidates who give well-reasoned points, not in the marking schedule, are awarded marks for doing so.

## B. Comments on *candidate performance in this diet of the examination.*

Most candidates were able to make a reasonable attempt at parts of each of the 6 questions. However, only a very limited number of candidates scored well in each of the questions.

In general, candidates demonstrated good knowledge of the core reading material and its application to familiar situations. However, in general candidates found challenging less familiar situations. This was evident particularly in question 3 and question 6 and resulted in lower scores from candidates this exam session. Additional comments are provided after each question below.

## C. Pass Mark

The Pass Mark for this exam was 55
41 presented themselves and 11 passed.


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**Solutions for Subject SP6 - April 2022**

**Q1**

(i)

A quanto option [½]
an insurance company balance sheet holding equities in a foreign currency can use quanto option to hedge the market risk of these holdings. [1]

A lookback option [½]
an insurance company might have guaranteed return products where the payoff depends on certain values that the stock market might have reached which might be hedged using a lookback option [1]

(ii)

Banks might engage in exotic options business because:
Margins are higher than for plain vanilla products [1]
Meet investor demand for more bespoke solutions [1]
Support wider business relationship [1]

Why some banks might not engage
More difficult, need staff and systems to support this [1]
Balance sheet impact can be more complex and higher [1]
Regulatory and collateral requirements might be tougher [1]
Miscommunication and reputational risks can be higher [1]
Activity is not core banking activity [1]
[Marks available 8, maximum 4]

(iii)

First we need to solve for gamma, we are short a binary cash-or-nothing call option so our Gamma is 0.0070. [1]
To offset this we need to sell call options 0.0070 / 0.0178 = 0.3933. (or purchase -0.3933 call options). [1]
Our resulting delta is -1.7104 - 0.3933 * 0.6248 = -1.9561
To be delta neutral we go long futures in the quantity 1.9561. [1]

(iv)

The hedge effectiveness of a hedge using futures and options is better compared to using futures only [1]
This is because the position can be delta and gamma hedged, instead of just delta hedged [1]
And therefore needs less frequent rebalancing [½]
Which is particularly important for hedging exotic derivatives, like binary options, where the delta can change rapidly [½]
The hedge is still quite approximate given the very different pay-off, delta and gamma exposures of a call option versus a binary option [½]
[Marks available 3½, maximum 3]

(v)

A cash-or-nothing call option can have delta in the range [0,inf] [1]
A cash-or-nothing call option can have gamma in the range [-inf,inf] [1]

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(vi)
The margin required for exchange traded derivatives is similar to the collateral required for bilaterally cleared derivatives, typically requiring daily valuation and IM. [1]

The bilateral collateral arrangements are specified in the contract between the two parties (in practice this is most commonly ISDA Master Agreement with CSA (Credit Support Annex) [1]

Whereas exchange traded margin is prescribed by the exchange [1]
This means that there might be more flexibility in the bilateral clearing [½]
Historically (IM) Initial Margin was less common for CSAs [½]
But now through new regulation (IM) is also required for bilateral clearing [½]
For bilateral arrangements the two parties need to value the contracts themselves [½]
And transfer collateral between each other [½]
Which means that for bilateral contracts the two parties will need to agree specifics around valuation agent, dispute resolution etc. [½]
The type of collateral might be different, e.g. whether cash, government bonds or other assets are eligible, bilaterally agreed or prescribed by the exchange [½]
Collateral arrangements might be more centralized and with less counterparties when using exchange traded, whereas bilateral might be with many different counterparties [½]
[Marks available 7, maximum 5]

(vii)
The bank could
align the bilateral terms to those prescribed by the exchange [1]
enter into other derivatives and manage the collateral on the overall book so that offsetting trades maximize collateral efficiency [1]
charge a premium for the binary option to make up for the cost of the increased collateral requirements [1]
limit collateral to cash only where possible [½]
repurchase assets received for cash to transform collateral [½]
absorb the impact of this as the profitability is sufficiently large or it might not be possible to pass costs to clients due to commercial pressure [½]
[Marks available 4, maximum 3]
**[Total 23]**

*Question 1 was reasonably well answered by most candidates, in particular the earlier parts of the question.*

*However, many candidates failed to generate a sufficient number of distinct points in parts (vi) and (vii) to pick up the majority of the marks.*

**Q2**
(i)
Present value of spreads is
(1.01^-1) * (1 - 0.03) * X +
(1.01^-2) * (1 - 0.03) * (1 - 0.03) * X +

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(1.01^-3) * (1 - 0.03) * (1 - 0.03) * (1 - 0.03) * X
(1.01^-4) * (1 - 0.03) * (1 - 0.03) * (1 - 0.03) * (1 - 0.03) * X
(1.01^-5) * (1 - 0.03) * (1 - 0.03) * (1 - 0.03) * (1 - 0.03) * (1 - 0.03) * X
= 4.4364 * X [1]

Present value of the default payments =
(1.01^-1) * 0.03 * ( 1 - 60%) +
(1.01^-2) * (1 - 0.03) * 0.03 * ( 1 - 60%) +
(1.01^-3) * (1 - 0.03) * (1 - 0.03) * 0.03 * ( 1 - 60%) +
(1.01^-4) * (1 - 0.03) * (1 - 0.03) * (1 - 0.03) * 0.03 * ( 1 - 60%) +
(1.01^-5) * (1 - 0.03) * (1 - 0.03) * (1 - 0.03) * (1 - 0.03) * 0.03 * ( 1 - 60%) +
= 0.05488 [1]
The CDS spread at which the CDS has zero value is where the present values offset each other and sum to zero [1]
Hence X = 0.05488 / 4.4364
X = 0.012371 or 1.2371% [1]

(ii)
The first-to-default will not trigger only if all bonds survive over the term of the contract. The probability of survival is (1 - 0.03)^5 for any bond and given no correlation the probability of 10 credits surviving (and the first-to-default not triggering) is ( (1 - 0.03)^5 )^10 = 0.2181 [2]
The probability of the first-to-default CDS triggering is therefore 1 - 0.2181 = 0.7819 or 78.19% [1]

(iii)
The first-to-default CDS value will decrease. [1]

With a higher level of correlation the conditionality of the 1-st to default option will become less valuable. The probability of the 1-st to default triggering is higher for a lower correlation and hence the value of the CDS. [1]

In the extreme of 100% correlation, the probability of survival for any bond is the same as for all bonds (1 - 0.03)^5 = 0.8587. The probability of the first-to-default CDS triggering is 1 - 0.8587 = 0.1413 or 14.13%. [1]
**[Total 9]**

*Many candidates scored well on parts (i) and (ii) and proved to be relatively straightforward for well-prepared candidates.*

*Most candidates found challenging part (iii) and were only generally able to score one of the marks available.*

**Q3**
(i)
When the individual payments of a bond are discounted by the appropriate zero-coupon spot yields the theoretical price of the bond is obtained. [1]

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If the same sequence of individual payments of a bond are discounted at a single interest rate to give the same theoretical market value, this single rate is the bond yield. [1]

(ii)
If a bond yield is negative, then it means that a bondholder will lose money if the bond is held to maturity. [1]

(iii)
At a basic level, negative interest rates mean that for vanilla interest rate options there will be negative strike prices. [½]
Also, prices of derivatives will be affected (in the same way as any change relating to interest rates). [½]
The impact on equity derivatives will be less than interest rate derivatives. [½]
There could still be an impact on equity derivatives, for example relating to the lower bound of an American option. [½]
In particular, the conditions to exercise American options early may be increased. [½]
From a modelling perspective some option pricing models may need updating to allow for negative interest rates. [½]
This is as a result of some models imposing a strictly positive assumption on interest rates. [½]
This will have a research and an implementation cost (through updating and testing systems). [½]
For some exotic options there may be additional complexities in the modelling. [½]
Market quotes for volatilities of interest rate derivatives may not exist. [½]
This then requires users of these derivatives to understand how market-quoted volatilities can be extended to negative strike prices. [½]
ISDA Master Agreements, in particular the CSA, may need clarification in respect of payment obligations under negative interest rates. [1]

[Marks available 6½, maximum 4]

(iv)
Let $C$ be the value of the vanilla call option and let the strike price of the option be $K$. [½]
In order to solve the stochastic differential equation in the more usual way (when $s = 0$), the adjusted forward price could be written as $F' = F + s$ and the strike price becomes $K' = K + s$. [½]
The volatility of this adjusted process is $\sigma_s$, in line with the specification of the model. [½]
Using the convention above the model becomes $dF' = \sigma_s F' dB_t$ and this is the more familiar stochastic differential equation with the following value for a vanilla call option. [½]

$$C = e^{-rT}\left[\left(F+s\right)N(d_1) - (K+s)N(d_2)\right],$$ [½]

where $T$ is the time to maturity in years, $r$ is the constant risk-free interest rate continuously compounded over the time to maturity and $N$ is the standard normal cumulative distribution function. [1]

$$d_1 = \frac{\log((F+s)/(K+s)) + 0.5\sigma_s^2 T}{\sigma_s\sqrt{T}},$$ [1]

$$d_2 = d_1 - \sigma_s\sqrt{T}.$$ [½]

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(v)
When $s = 0$ this becomes Black's model and $F$ is drawn from a lognormal distribution. [½]
The lowest value $F$ can take is 0. [½]

(vi)
This model is able to allow for negative forward rates. [½]
The value of $s$ determines the lowest value which forward rates are able to take. [½]
As discussed in part (v), when $s = 0$ the model cannot accept negative rates but when $s > 0$ the lowest value which the forward rates can take is $-s$. [1]
A sufficiently large value of $s$ is required so that $F + s$ is a positive value for all forward rates implied. [1]
This value needs to be selected before the model is run. [½]
One problem with this model is determining a suitable level of volatility $\sigma_s$. [½]
In general this is not equal $\sigma_0 + s$. [½]
Instead, a calibration to option prices is required. [½]
The model has a closed analytical formula for standard options. [½]
Due to the similarities with Black's model, the same methods of analysis can be used with this model. [½]
This extends to easily adapting existing IT systems or existing software. [½]
[Marks available 6½, maximum 4]

(vii)
In general, not being able to model negative interest rates is going to lead to increased model risk. [½]
The level of this risk will depend on the exposure to negative interest rates (for example if the firm has no interest options there is a low model risk) [½]
and the level of negativity (for interest rates negatively close to zero, say 10 basis points, then the risk is lower than having larger negative interest rates, say 100 basis point). [½]
From a modelling perspective it will be impossible to reproduce the spot interest rate curve. [1]
As an example, this could be significant in Solvency II calculations, [½]
or in the production of scenarios from an economic scenario generator. [½]
If the financial institution has exposure to the cost of options and guarantees then these could be underestimated. [½]
Any form of generating discount factors used for discounting cash-flows would be higher than expected. [½]
[Marks available 4½, maximum 3]
**[Total 20]**

*Candidates found challenging this question considering negative interest rates have been a feature of fixed income markets for a number of years.*

*Candidates performed very well in parts (i) and (ii) but found challenging the remaining parts of the question.*

**Q4**

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(i)
Defined Benefit
Value at risk ("VaR") is useful for pension scheme trustees to assess the amount of risk exposure their investment managers are taking in order to achieve the returns on the portfolio. [1]
Being able to measure risk adequately is important for a sponsoring employer to understand as the level of future contributions will depend on investment performance (and implicitly the risk within the investment portfolio), as well as other factors. [1]
Due to the simplicity of VaR it can not only be used on the asset position but also in analysing the liabilities. [½]
It has the additional benefit in being able to be decomposed into specific components, both for liabilities and assets. [½]
Hence it can be used VaR to quantify the additional deficit which could occur over a period and with a certain level of probability. In this way it captures the overall financial risk of the scheme. [1]
Pension schemes also use derivatives, for example swaps, and VaR can be used in establishing collateral requirements.

Defined Contribution
For individuals VaR may be useful in understanding their risk for making portfolio decisions, [½]
as these decisions will have a direct impact on the level of their retirement pension. [½]

General
It may be a requirement of regulators or legislation to use this measure to disclose information. [½]
Similarly, it may be required for tax, audit or accounting purposes. [½]
[Marks available 6, maximum 4]

(ii)
Like any risk measure the use of VaR depends on the quality of the inputs, which can be difficult to establish. [½]
There are choices to pension schemes about what level of VaR to choose: 90, 95 or something else. [½]
VaR can be used to determine the percentage of the time the outcomes will be worse Than the calculated VaR values, it is unable to quantify just how much worse. [1]
There could be extreme losses in the tails which could lead to some extreme funding positions. [½]
For trustees and individuals there may be a barrier to understanding VaR and how it is used. [½]
Historic calculation of VaR cannot be used in optimization problems relating to the pension scheme portfolio. [½]
[Marks available 3½, maximum 3]

(iii)
Using the equation given in the question: $\delta P = 45\delta S$ . [½]
The change in the return of the underlying is: $\delta r = \delta S / S$ , so $\delta P = (1350 \times 3)\delta r$ and $\delta P = 4050\delta r$ . [½]
Let X be the required VaR. From the definition of VaR, $\text{Prob}(\delta P \le -X) = 0.05$ . [1]

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From the information provided, $\delta P \sim N(0,(4050\times 0.005)^2) = N(0,20.25^2)$. [1]
Using the inverse normal function, $N(-33.308) = 0.05$. [½]
Hence, 95% VaR = 33.308. [½]

(iv)
Due to the complexity of a pension scheme and the number of risk variables a Monte Carlo based approach would be appropriate. [1]
Other methods may be more approximate and not reliably estimate VaR. [½]
Many pension schemes also make use of derivatives, and due to potentially large gammas, Monte Carlo methods are appropriate. [1]
This includes non-linear instruments and path-dependent instruments. [½]
Monte Carlo methods can also deal with many asset classes in a computational efficient way. [½]

[Marks available 3½, maximum 2]

**[Total 13]**

*Question 4 was reasonably well answered by candidates. The top performing candidates were able to generate a sufficient number of distinct points and typically scored well in the early parts of the question.*

*Candidates less prepared often provide generic information on VaR metrics and approaches which was not tailored to the specifics of the question and therefore were unable to achieve full marks.*

**Q5**

(i)
The control variate technique assumes that the error inherent in the use of a tree is the same for American options as for European options. [1]
The tree is firstly used to calculate the price of the American option being valued and the same tree is then used to calculate the value of an equivalent European option. [1]
The analytical solution for the European option is then calculated and the estimate of the model error defined to be the difference between the price calculated analytically and that calculated using the tree. [1]
The error term is then added to the calculated value of the American option. [½]

[Marks available 3½, maximum 3]

(ii)
Using, t=1, r=3%, S=130, K=135, q=2% and σ=40%, the BS model gives the price of the European put option to be 22.324
d1 = 0.13065
d2 = -0.2694
N(-d1) = 0.44803
N(-d2) = 0.60617 [2]
p = K x exp(-r*t) x N(-d2) - S x N(-d1) x exp(-q*t)
= 135 x 0.9704 x 0.60617 - 130 x 0.44803 x 0.9802
= 22.324 [2]
The tree prices the asset as follows

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u = exp (σ x sqrt(½)) = 1.3269
d = exp (-σ x sqrt (½)) = 0.75364a = exp ((0.03-0.02) /2) = 1.00501
p = 0.4385
1-p = ½615 [2]
This gives the following tree

| t | 0.00 | 0.50 | 1.00 |
|---|---|---|---|
| | | | 228.89 |
| | | 172.50 | |
| | 130.00 | | 130.00 |
| | | 97.97 | |
| | | | 73.84 |

Therefore,
a = 130.00
b = 172.50
c = 97.97
d = 228.89
e = 130.00
f = 73.84 [2]

The European option is priced by working backwards through the tree

**European**

| t | 0.00 | 0.50 | 1.00 |
|---|---|---|---|
| | | | 0.00 |
| | | 2.77 | |
| | 21.10 | | 5.00 |
| | | 35.99 | |
| | | | 61.16 |

[2]

a = 21.10
b = 2.77
c = 35.99
d = 0
e = 5.00
f = 61.16

Similarly, the American option is priced by working backwards through the tree but substituting any node value where it would have been optimal to exercise

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| American | | | |
|---|---|---|---|
| t | 0.00 | 0.50 | 1.00 |
| | | | 0.00 |
| | | 2.77 | |
| | 21.68 | | 5.00 |
| | | 37.03 | |
| | | | 61.16 |

a = 21.68
b = 2.77
c = 37.03
d = 0
e = 5
f = 61.16

[2]

The American option is therefore valued as 21.68+(22.324-21.10) = 22.90 [1]

[Marks available 13, maximum 8]

(iii)
To calculate the value of a derivative, Monte Carlo techniques simulate many sample paths of S, the underlying asset, under the assumption of risk neutrality. [½]
The value of the derivative is then calculated as the mean of the expected pay-offs, discounted under the risk-free curve. [½]
As the value of the asset is known along each sample path, the process is suitable for valuing path-dependant options, such as Asian options [1]
American options enable the investor to exercise the option at any (or specific) times prior to the expiry date. When valuing these options, it is necessary to know if it is optimal to exercise the option early. [1]

As noted above, under the Monte Carlo approach, the value of the derivative is an average of the derivative payoffs.
Therefore, the assessment of whether it is optimal to exercise the option early under each sample path is contingent on the other sample paths which creates a significant degree of complexity. [1]
On the other hand, trees enable the direct comparison of the early payoffs and allow (relatively) straightforward assessment of whether it is optimal to exercise an option early, making them suitable for American options. [1]

[Marks available 5, maximum 3]

(iv)
The Longstaff Schwartz least squares approach defines a relationship between the sample paths which can be used to value American options in a Monte Carlo simulation. [1]

**[Total 15]**

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*Candidates performed very well in this question. Most candidates were comfortable doing the required calculations and also scored well on the qualitative parts of the question.*

**Q6**

(i)

To price currency options, we need to take into account the risk-free interest that can be earned on the foreign currency during the life of the option. [1]

This can be achieved by reducing (or increasing in a negative interest rate environment) the amount of the currency being valued. [½]

If the foreign risk-free interest rate is $r_f$, then the value of the foreign currency $S_o$, in the BS pricing formulae is substituted by $S_o e^{-r_f T}$. [½]

Forward rates, F0, implicitly account for the risk free interest which is earned in the foreign currency and can be used to further simplify the pricing formulae. [½]

This is the same approach which is taken to valuing stocks with a known dividend yield. The risk free interest rate is analogous to the dividend yield [½]

[Marks available 3, maximum 2]

(ii)

Using the information provided, we can set up the two sales and profit projections

| | Units | Retail price | | Sales revenue | | Cost (USD) | |
|---|---|---|---|---|---|---|---|
| **Sc1** | 20000 | £ | 40 | £ | 800 000 | $ | 1 000 000 |
| **Sc 2** | 70000 | £ | 35 | £ | 2 450 000 | $ | 3 500 000 |

[2]

We require profit to be £100k, therefore

Min profit = Revenue - (Cost of goods ($) x Strike) - Option premium (£) [1]

Using the information in the question, we have

| | |
|---|---|
| r | 2% |
| rf | 3% |
| t | 0.50 |
| S | 0.65 |
| σ | 20% |

[1]

a) Scenario 1

The cost of call option for $1 is

= ((0.65 x exp(-0.03*½) x 0.429) - (0.67 x exp(-0.02 x ½) x 0.0.374

= 0.0264 [2]

Therefore,

Minimum profit = 800,000 - (670,000) - (26,400) = £103,600 [1]

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b) Scenario 2
= ((0.65 x exp(-0.03*½) x 0.429) - (0.67 x exp(-0.02 x ½) x 0.0.374
=0.0502 [2]
Minimum profit = 2,450,000 - (2,170,000)- (3,500,000 x 0.0502)= £104,400 [1]
[Marks available 10, maximum 7]

(iii)
If the exchange rate is above USD:GBP 0.67, then both contracts generate a profit in excess of £100,000 when hedging with the options. [1]
If the exchange rate is between 0.60 and 0.67, then purchasing 20,000 units (Scenario 1) generates a higher profit. [1]
If the exchange rate is lower than 0.60, then purchasing 70,000 units (Scenario 2) would provide the better payoff. [1]
Therefore, if the finance director does not expect much deviation in the exchange rate, she should opt to purchase the lower quantity. [½]
However, her proposal to use options to protect against exchange rate risk suggests she thinks there could be some volatility in the next six month…. [½]
and therefore she may opt to order 70,000 with the aim of benefitting from a depreciation in USD while still achieving her objective of a minimum profit of £100k. [½]
[Marks available 4½, maximum 3]

(iv)
A bull call spread consists of the purchase of a call option coupled with the sale of a call option with a higher strike price. [1]
Provided the notional amounts and term to maturity are the same, this creates a payoff within a given range, if the contract expires in the money [½]
In this case, the retailer could sell a call spread which would give an option structure which ensured that the profit was always between the desired range of £100-200K [1]
[Marks available 2½, maximum 2]

(v)
Using the same parameters as before, the option premium from a call at USD:GBP 0.70 is 0.0169. [1]
Therefore the option premium are as follows
Sc1: £26,500 - (1,000,000 x 0.169) = £9,400 (rounded) [1]
Sc2 £175,555 - (3,500,000 x 0.169) = £59,300 (rounded) [1]

(vi)
By creating a bull call spread, the profit from Scenario 2 is usually greater if the exchange rate is below 0.7 (except for small point around. [1]
However, if it is above 0.75, then there will be a loss generated in Scenario 2. [1]
This is not the case for scenario 1 where a profit is generated. [½]
Therefore, if the finance director was convinced by the output of the research, then she should order 70,000 units. [½]
However, if she was risk adverse then she should opt to order 20,000 units. [1]
[Marks available 3½, maximum 3]
**[Total 20]**

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*Many candidates were able to demonstrate good knowledge of the option pricing calculations and to use this in the given scenario and scored well in the calculation parts of the question.*

*Only very well prepared candidates were able to score points in other parts of the question.*

**[Paper Total 100]**

# END OF EXAMINERS' REPORT


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