PENSION FUND VALUATIONS AND MARKET VALUES

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ABSTRACT

The traditional approach to United Kingdom pension fund valuations is to use an off-market approach to valuing assets and liabilities. This approach has been called into question for a number of reasons, such as changes to the taxation of U.K. share dividends and a growing understanding and appreciation of the key principles of financial economics. This paper looks at the history of the traditional approach and focuses on the drivers for change. We compare the properties of various methods that take assets into the balance sheet at market value against the traditional valuation method. Our principal aim throughout has been to produce a paper that is practical and helpful to pension scheme actuaries.

KEYWORDS

Pension Schemes; Valuation Methods; Market Values; Funding; Pricing

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1. INTRODUCTION

1.1 In the United Kingdom, the traditional approach to the actuarial valuation of a defined benefit pension scheme has been to take assets into account at a value other than market value. This has generally been determined as the present value of the expected future income stream, predominantly dividends from equities.

1.2 In 1997 the Chancellor of the Exchequer decided to withdraw U.K. pension schemes' ability to reclaim the Advanced Corporation Tax (ACT) credit on U.K. company dividends. U.K. pension schemes, thereafter, were only entitled to receive dividends net of tax, a difference worth up to 20 pence per £1 of gross dividend. This substantial change in the taxation of U.K. dividends implied, in the absence of remedy, a reduction of up to 20% in the value placed by actuaries on the assets of a pension scheme, with knock-on effects for disclosed funding levels and contribution rates. In the face of such a significant impact from taxation and a number of other factors driving changes to the U.K. pension scheme environment, actuaries began to question the merits of traditional asset valuation methodologies.

1.3 In response to this, the Technical Support and Research Committee of the Pensions Board established a Working Party to consider the use of market values for actuarial valuations. This is the report of that Working Party.

1.4 The terms of reference set by the Technical Support and Research Committee were as follows:

(1) to consider the merits of various valuation methodologies for U.K. pension funds following the ACT change; in particular, to consider how the assets should be brought into the actuarial valuation balance sheet;
(2) to assist the MFR Change of Conditions Working Party in formulating any changes to MFR methodology; and
(3) to have regard to:
  — work being carried out by the International Accounting Standards Board;
  — current practice;
  — the needs of users of valuations; and
  — the importance of effective communication by the profession.

1.5 In interpreting these terms of reference, the Working Party believes that the needs of the profession are best served by the following:

— to describe the different methods of producing a valuation where assets are taken into account at market value;
— to compare these methods against a suitable set of criteria;
— to compare these methods against the traditional assessed value methodology;
— to produce a paper that is practical and helpful to pension scheme actuaries; and
— to produce a paper written in a language familiar to pension scheme actuaries.

1.6 The main body of the paper is laid out as follows. In Section 2 we examine the purposes for which actuarial calculations may be required. Section 3 looks at the history of how we got to where we are today. Section 4 describes the drivers for change from current practice, with Section 5 expanding on the key principles of financial economics as they relate to pension liabilities. In Section 6 we describe alternative methods of conducting a valuation, and Section 7 introduces a common set of criteria, or properties, for comparing those methods. Section 8 shows the results of modelling the behaviour of the different methods on the funding level and contribution rate of an example scheme. In Section 9 we compare these methods against the properties described in Section 7, and in Section 10 we draw our conclusions.

2. The Purpose of an Actuarial Valuation

2.1 Before comparing different valuation methods, it is worthwhile to have a clear view of their purpose.

2.2 There are four principal subsets of purposes underlying actuarial valuations: funding; commercial transactions; accounting and regulatory.

2.2.1 *Funding*

In general terms, a client may wish to know one of four things:

(a) the future contribution rate;
(b) how far the assets cover accrued liabilities, allowing for future pay increases;
(c) how far the assets cover accrued liabilities, were the scheme to be wound up and the benefits secured with an appropriate pension provider; and
(d) how a disclosed surplus (or deficiency) should be allocated between different classes of member or employer.

2.2.2 *Commercial transactions*

In this context a transaction is taken as a cash payment in respect of retirement benefits. These situations cover individual or grouped (bulk) payments, e.g. transfer payments. Individual cases might also include, say, special retirement options or augmentation payments. Setting up or changing pension arrangements also represents, in principle, a transaction between employer and employees.

2.2.3 *Accounting*

Sponsoring employers require pension expense calculations for their company accounts. More than one set of calculations may be required because of different accounting principles in the U.K. (SSAP 24) and elsewhere internationally (e.g. FAS 87 for United States reporting).

2.2.4 *Regulatory*

Valuations are required to comply with regulatory standards, most notably the Minimum Funding Requirement (MFR), demonstrating the ability to contract out of SERPS and testing for excess surplus.

2.3 The traditional actuarial valuation of a pension fund is primarily concerned with setting a contribution rate, namely under ¶2.2.1(a). This is part of a procedure for controlling the pace at which a fund is built up to meet the liabilities, and involves a number of assumptions. In many ways it can be considered as an algorithm for setting a contribution rate, but with the assumptions used determining the pace of funding.

2.4 In a traditional valuation, it is conventional to work with present values rather than rolling all payments up to a terminal date. The use of a constant interest rate for all time periods is not essential for accumulation or discounting, but, conveniently, does allow the use of standard actuarial commutation functions to switch back and forth. It is when we work with present values that confusion can, however, emerge in terms of the meaning of the resulting numbers. Although they may appear superficially similar, these present values are not ‘values’ in the sense typically used for commercial transactions or corporate finance.

2.5 The historical development of traditional actuarial valuation techniques is described in Section 3. In the analysis of different methods which follows it is termed Method 0 (but Method 1 is very closely related to it).

2.6 As the other purposes of actuarial valuations have appeared, generally it has been the traditional method which has been adapted and used (with the exception of FAS 87, which prescribes a different approach).

2.7 In more recent years, a different approach to considering pension scheme liabilities has developed, based on the concept of market pricing. This is based on the wide body of literature and theory associated with financial economics and corporate finance, a potted history of which is given in Section 5.

2.8 In this paper this market pricing approach of financial economics is termed Method 3, or the 'economic valuation'.

2.9 Neither the income from the assets held nor an assumed return on net inflows or outflows enter an economic valuation. In fact, the discount rates used to value the liabilities have a subtly different role. It is misleading to think of the discount rates used to value liabilities for a transaction in terms of an assumed future return on assets, it is far clearer to think in terms of forward rates and their direct link with today's asset prices.

2.10 Understanding this difference between a traditional valuation and an economic valuation is fundamental to the sections which follow.

2.11 It is sometimes suggested that these two valuation approaches can be used interchangeably. In reality they perform distinctly separate roles, and some basic financial parameters require subtly different interpretation. For example, since expected returns on new money have no real role in valuations within financial economics (as described below), a modification of the financial economic approach for setting a contribution rate, by allowing for anticipated returns on a 'fund', requires care in presentation. Equally, the returns in a traditional valuation are supposed to reflect returns on actual assets held, so use of expected returns on notional matching portfolios can also be somewhat confusing. The main difficulties which lie ahead in any discussion of pension fund valuations and market values are thus largely concerned with disentangling attempts to merge the distinct approach of the traditional valuation and the financial economics approach of market pricing. We defer discussion of various merged approaches to later sections.

3. The Historical Development of the Traditional Valuation Method

3.1 In this section we examine the background to, and historical development of, the traditional methods used in pension fund valuations. The purpose of this is to explain how and why current practice developed, before moving on, in Section 4, to look at the factors which are now causing many actuaries to consider departing from the traditional approach.

3.2 *Actuarial Valuation of Pension Fund Liabilities*

3.2.1 Up until the 1970s pension fund valuations were primarily concerned with setting a contribution rate. This was frequently determined by the aggregate funding method, as the value of all future liabilities less the value of assets spread over the future lifetime of the fund. The entry age method was also in common usage, though the recommended contribution rate also involved quantification of both the above values.

3.2.2 As discussed in the previous section, in order to establish a contribution rate some assumption is required as to the return on investments, and this is traditionally dealt with by accumulating or discounting liabilities expected at future times to a single date, using an assumed return on assets. This naturally led to the publishing of a figure representing the 'present value' of scheme liabilities. Whilst this was actually only a mechanical calculation on a subjective set of assumptions, it did come to be viewed as the actuary's estimate of the value of the scheme's liabilities in its widest sense.

3.2.3 The textbook by Lee (1986) showed that, in order for the result to be the same regardless of the chosen date for discounting values, it was necessary that the valuation rate of interest should be equal to the average rate of interest at which existing assets and future contributions were assumed to be invested in future. If a different discount rate was chosen, then, even if experience was exactly as expected, the derived contributions would accumulate over time to an amount that was different to that of benefits paid. The long-term rate of interest was, therefore, an amalgam of *current* investment returns available in the market and *future* unknown investment returns. The crucial point is again that, in order to derive a contribution rate, the actuary was required to make an assumption about future long-term investment returns.

3.2.4 Lee's work was by no means original in this respect. Earlier textbooks by Porteus (1946) and Crabbe & Poyser (1953) reached the same conclusion. The contemporary practice for combining the *current* and *future* investment return assumptions into one valuation rate of interest was to weight the two components respectively by the size of the existing fund versus the annual sums to be invested in the future. As most schemes were immature, most attention was focused on the latter rather than on the former.

3.2.5 Earlier, Puckridge (1947) had advocated a simpler approach for the valuation rate of interest, i.e. to consider only the long-term rate of interest that could be earned on *future* investments. The key proviso for this simplification is that assets have to be brought into the balance sheet by discounting *future* income, regardless of whether this is derived from existing assets or new assets held in the future, at the same rate of interest. Since the concept of discounting income from assets had not been developed, Puckridge initially received little support for his work. Only when more attention was focused on the actuarial valuation of pension fund assets did his work receive wider acceptance.

3.2.6 The process of determining the numerical value for the long-term valuation rate of interest developed by observing stable historic differences between investment returns and price inflation. In the early twentieth century it was not necessary to fix an assumption for price inflation, since this was assumed to be neatly offset by the growth in liabilities through pay escalation and pension increases. As Lee said, even as U.K. economic conditions became less stable in the second half on the twentieth century, the financial stability of pension schemes showed some resilience. Taking one year with another, investment performance in excess of the assumed real return of 3% or 4% helped to cover the additional liabilities arising from pay increases greater than expected and to provide resources from which discretionary pension increases could be granted.

3.3 *Actuarial Valuation of Pension Fund Assets*

3.3.1 The assessment of pension fund liabilities using a long-term return on assets to discount the expected cash flows, therefore, received a good deal of attention in earlier literature. Until the 1960s, however, the assessment of the value of assets to input into the contribution rate calculation had not. At that time it was still conventional to value assets at book value (or market value if less). Book values are merely an accounting measure to value individual investments according to the date of their purchase. This produces the odd result that identical investments are attributed different values in the same scheme depending on when they were purchased.

3.3.2 It was this fact that led Day & McKelvey (1964), following the earlier work of Heywood & Lander (1961), to develop their dividend discount model for valuing equity assets. This is confirmed by K. J. McKelvey, the author's son, who recalled why his father developed an off-market valuation basis for assets:

> ".. the sole objective of that 1964 paper was to find a consistent basis for valuing assets, given an existing methodology for valuing liabilities. The liability valuation basis was off-market, by convention, at that time. Therefore, the asset valuation inevitably became off- market. The main aim of the authors was to move away from the valuation of assets by book value, which was still common. They simply did not think about market values since most pension schemes were new and immature and there were no formal discontinuance tests and the like".

3.3.3 Indeed, references in the paper to market values were generally limited to explaining their inappropriateness when compared with an actuarially assessed value of liabilities. In the ensuing discussion of the paper, only one person, Mr J. Plymen, considered the idea of a consistent market-related valuation of both assets and liabilities. In today's context, his comments are most interesting. To quote from the *Journal*:

> "Mr J. Plymen felt that the impression was getting around that there was something quite immoral in valuing assets at market values, whereas an elaborate valuation process…that produced a figure [of assets] 20% higher than the market value was perfectly respectable! He felt that that was tackling the problem from the wrong end. It was traditional that with a life office valuation a decision was taken on the rate of interest for the valuation of the liabilities and that was that. The authors were taking the same line with a pension fund valuation. They were assuming a certain figure for the rate of interest for valuing the liabilities, and twisting the valuation of the assets round to be consistent with that basis. Why not start off with the

market value of the assets and try to deduce from that basis a consistent system for valuing the liabilities?”

3.3.4 Nevertheless, Day & McKelvey’s work was such an improvement on what had existed before that it became the widely accepted funding methodology by nearly all U.K. pension fund actuaries. Perhaps its worth was most appreciated following the 1974/75 Stock Market collapse. The fact that assessed values were being used to measure assets meant that plan sponsors were not suddenly required to increase the contribution rate into their pension schemes. It was this implicit smoothing mechanism that helped to make the methodology so popular.

3.4 *Consistency*

3.4.1 The historical development of valuing U.K. pension fund assets and liabilities led to the concept of ‘consistency’ within actuarial pension fund work. What most actuaries mean by this is that the unit of currency is the same on both sides of the balance sheet. The concept arises because our historical development provides us with a choice in presenting valuation results: either

(a) an actuarially assessed value of assets compared with an actuarially assessed value of liabilities; or
(b) market value of assets compared with a market-related value of liabilities.

3.4.2 The U.K. actuarial approach has developed along the lines of (a), with significant interest only being placed on (b) within recent years. Had we not developed the off-market valuation approach in (a), then this consistent currency concept would not have risen to such axiomatic prominence.

3.5 *Adoption of Accrued Benefits Funding Methods*

3.5.1 The funding methods widely used in Day & McKelvey’s era are examples of prospective benefits funding methods. These methods generate a contribution rate, firstly by evaluating total service benefits, rather than a deliberate separation of past and future service liabilities. In the case of the aggregate method, past service surpluses or deficits are implicitly spread over the average remaining working lifetime of active members.

3.5.2 In the last thirty years the rising maturity of pension schemes has increased the importance of past service values. Today the projected unit method dominates pension scheme funding, and, as an example of an accrued benefits funding method, specifically focuses on the quantification of past service liabilities. The reason for this is that, prior to the introduction of the Minimum Funding Requirement, the ongoing funding level became known as the prime measure of the financial security of members’ accrued benefit expectations.

3.5.3 The shift towards accrued benefit funding methods is one example of how pension fund valuations have changed over the last thirty years. Further drivers for change are considered in Section 4.

4. Drivers for Change

4.1 *Success to Date*

4.1.1 The traditional approach (i.e. whereby an assessed value of assets is compared against a value of liabilities determined using a long-term rate of return) has succeeded in meeting the following objectives over the last thirty years or so:

— acceptably smooth past service funding levels, reducing the volatility in the market value of U.K. equity assets;
— acceptably smooth and stable future service contribution rates; and
— presentational credibility, with general acceptance of the underlying theory by the various parties traditionally involved.

4.1.2 However, the conditions during which these objectives have been met are characterised by:

— stable dividend policy by U.K. companies;
— two major periods of market volatility (a sharp dip in 1974 and a sharp spike in 1987), which enhanced the credibility of the approach;
— the success of a U.K. equity biased investment strategy;
— high levels of discontinuance solvency arising from greater pre-funding of discretionary benefits and lower early leaver entitlements than is the case today;
— a relatively low level of concern about the risks inherent in different investment strategies among trustees and company representatives traditionally involved in pensions management;
— even after the introduction of SSAP24, significant flexibility in the way valuation results are presented by company management in financial statements; and
— a requirement for trustees to monitor the funding position of their scheme only once every three years.

4.1.3 In this section we first consider, in turn, the three features of the traditional method which have, in our opinion, defined its past success, and discuss the forces which are currently casting doubt on its future. Finally, we address some of the key external forces for change.

4.2 *Smoothness of Funding Levels*

4.2.1 As noted above, one of the major features of the traditional approach is that it smoothes out short-term fluctuations in market values. This effect can be seen in Figure 4.1.

4.2.2 This smoothness arises from the historic stability of U.K. dividends. This historic effect is not in question. If a smooth series is divided by a constant yield, then the result is an equally smooth series of actuarial values, provided that the assets are invested in the U.K. equities which underlie this calculation.

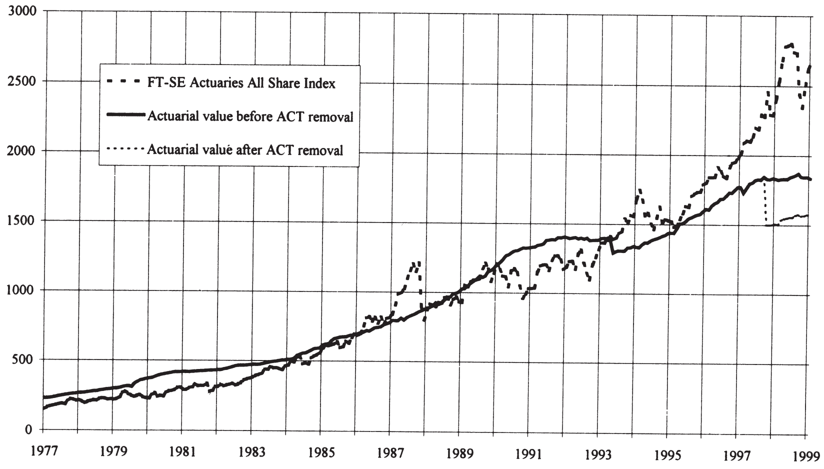

Figure 4.1. Market value vs actuarial value

4.2.3 The long-term smoothness of the method applied in this way to U.K. equity investment will only be threatened if dividend payments become more volatile in the U.K. Step changes in taxation aside, the main driver here would be a change in the attitude of U.K. company management towards dividends.

4.2.4 It has been common, historically in the U.K., for company management to see dividend stability (and growth) as a business objective. Thus, dividends may have been paid even when capital was being raised, and shareholder funds may have been retained even when no immediate investment opportunities presented themselves. This has been an important factor in the historic stability of U.K. dividends, and hence the smoothness of asset values under the traditional method. In the U.S.A., by contrast, it is more commonplace for companies to distribute funds to shareholders only when all the investment opportunities have been utilised, and, on the other hand, to make large repayments (typically through share buy backs) rather than retain shareholder funds. Under this U.S. model, which is more consistent with the modern business objective of enhancing shareholder value, dividend series are, therefore, far less stable. There is some evidence emerging for changes in U.K. payment patterns.

4.2.5 Another feature, particularly relevant to the MFR, is the dependence of this smoothing process upon the choice of notional portfolio. The traditional approach gives smoothness only if the notional asset portfolio is closely linked to the actual assets held. In reality, most schemes have historically used a notional portfolio which assumed a relatively high U.K. equity content, whilst the scheme has, perhaps, only held around 60% of its assets in U.K. equities. The effect of this is to create more volatile past service funding ratios than would be anticipated if the notional and actual portfolios coincided. The best example of

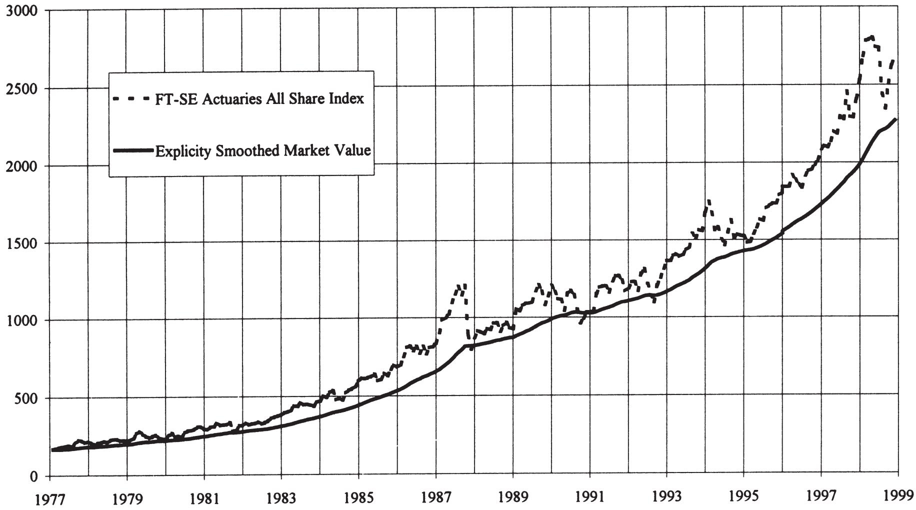

Figure 4.2. Market value vs smoothed market value

this is to consider a major correction in U.K. equities (e.g. October 1987) when, instead of protecting the scheme's funding position, the effect of using the typical traditional valuation basis assuming 100% investment in U.K. equities would have been to increase the funding ratio. More recently, divergence between U.K. and overseas equity returns has led to some instability in MFR calculations based on U.K. equity notional funds. One solution is to move the notional portfolio closer to the actual assets being held by the scheme. However, as schemes invest more in overseas asset classes, where yields are perhaps more volatile and reliable statistics are harder to find, this notional portfolio problem becomes harder to solve, and attempts to apply the traditional method to such assets appear to have been unsuccessful.

4.3 *Smoothness of Future Service Contribution Rates*

4.3.1 Future service contribution rates under the traditional method appear to be smooth, since the same long-term investment assumptions are applied at each valuation, therefore contribution rates change only with changes in demographics and benefits. By contrast, significant volatility would be introduced by any changes to long-term investment assumptions, due to the long duration of new accrual.

4.3.2 However, even if long-term investment assumptions are kept constant over time (and, in practice, this may be unlikely), this stability actually arises from an inconsistency in the traditional method. This inconsistency is revealed, for example, if we consider an immature scheme when actuarial values are above market values, and where the employer pays the contributions recommended by the actuary. We find that surplus emerges even if all the actuary's assumptions

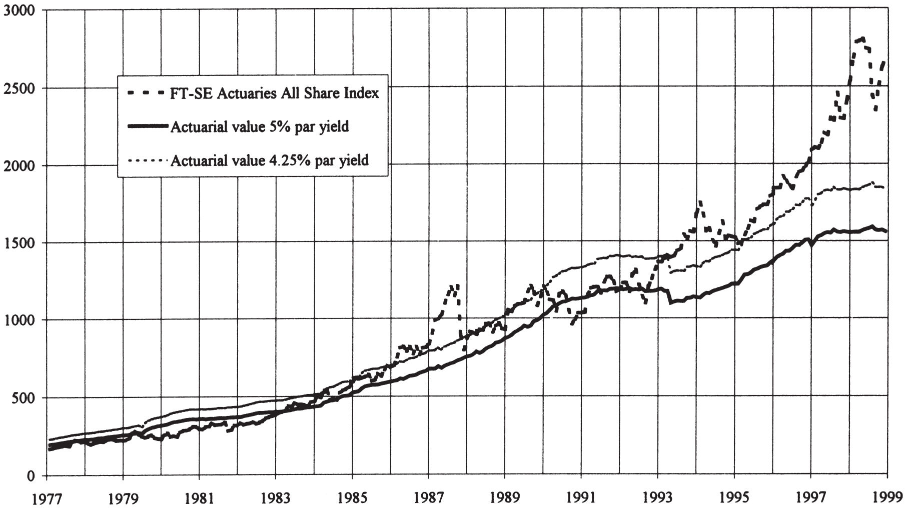

Figure 4.3. Market value vs actuarial value; impact of changing par yield

were borne out in practice. This effect arises, of course, because consistent application of the traditional method would also apply the asset adjustment to the derived contribution rate — thus producing lower contributions in this case. Ironically though, applying the traditional method consistently in this way can result in a highly volatile overall contribution rate, which negates the efforts made to smooth the other components in the calculation.

4.4 *Credibility*

4.4.1 If we return to Figure 4.1, we note that, although the historic smoothness of U.K. dividends (step changes in tax aside) is not in doubt, the vertical positioning of the dashed line is entirely dependent on the choice of divisor. If the constant average yield is increased by 25% (from 4% to 5%, say), then all values reduce by 20%.

4.4.2 Dyson & Exley (1995) pointed out that an equally smooth (or smoother or less smooth, if required) series of values could be obtained by any explicit smoothing technique. The solid line in Figure 4.2 shows one particular method, their geometric method. The credibility underlying the explanation of the traditional approach, when compared with an arbitrary choice of smoothing method, hinges on a supposition that dividend yields will revert to a single long-term average yield over time.

4.4.3 There have been historical events which appear to support this 'mean reversion' effect, most notably in 1974 and 1987. However, there also appear to be secular trends in dividend yields, which make the long-term average itself difficult to establish going forward (although easy to establish with hindsight).

4.4.4 A secular change in dividend yields can leave the traditional method generating values constantly well below (or well above) market values. Figure 4.3 illustrates this. This shows the traditional method based on the long-term yield of 5% which was prevalent in the early 1980s. Arbitrary step changes are required in the assumed long-term yield to address this, and experience of such changes has been one factor undermining the traditional approach used for the MFR.

4.4.5 The choice of long-term dividend yield is therefore important to the presentational credibility of the traditional method. This explains why the removal of the ACT tax credit enjoyed by pension schemes on U.K. equity dividends acted as such a catalyst for reappraisal. As a consequence of the change, it is expected that companies will follow the U.S. example of using other more tax efficient means (such as share buy backs) to return funds to shareholders (this trend was, in fact, already under way before the 1997 Budget). This makes the appropriate long run average equity yield difficult to predict with credibility. Without credibility in this number, assessed asset values could be placed in a wide range, depending on how future investment returns are assumed to be divided between income yields and dividend growth.

4.4.6 It has often been argued that there is a stable relationship between dividend growth and salary inflation. If a strong and stable link existed between these two factors, then, for example, errors in the assumed rate of dividend growth would be offset by errors in the assumed rate of salary growth. Without this link, valuation results are highly sensitive to the assumed division between equity income yield and growth. A number of papers (for example, Dyson & Exley, 1995; Exley, Mehta & Smith, 1997; Smith, 1998) have directly challenged the existence of this link between equities and salary-related liabilities, and argued that the statistical evidence for any such link is very weak.

4.5 *External Forces*

4.5.1 Finally, there have been three main external forces challenging the use of traditional actuarial valuation methods in recent years.

4.5.2 There is a growing understanding and appreciation within the actuarial and accounting profession of the key principles of financial economics. Furthermore, these principles are increasingly taught in management schools, thereby increasing potential client exposure to the concepts. This theme is advanced in Exley, Mehta & Smith (1997), which goes on to explain how, in many applications, market values are the right measure to use from the shareholder’s perspective, even if markets themselves are inefficient and market values do not reflect rational expectations. However, this approach can lead to conflict between the desires of shareholders for transparent and objective valuation methods and the interests of other stakeholders. Management of this conflict represents a challenge for any valuation method.

4.5.3 Some of the key principles of financial economics are explained in Section 5. This goes on to discuss some of the practical issues surrounding their application to pension liabilities.

4.5.4 Secondly, the Government imposed a Minimum Funding Requirement (MFR), with effect from April 1997 (subject to transitional arrangements). The existence of an MFR introduces short-term funding considerations, since a drop in a scheme's MFR funding level can result in the employer having to pay in significant contributions at short notice. This has led to a desire by trustees and employers to gain a greater understanding of the factors affecting a scheme's financial position. Since the majority of active and deferred pensioner liabilities are effectively valued using the traditional method (linked to U.K. equity yields), this has focused clients' attention on many of the issues described above.

4.5.5 Finally, the new international accounting standard, IAS19, uses market-based methods, and this has prompted the U.K.'s Accounting Standards Board (ASB) to consider market related methods for the revision of SSAP24. This move again appears to be motivated by the desire for transparency, conflicting with the smoothing sought from the traditional method, which potentially removes a lot of information regarding short-term effects on the financial position of the scheme. This may be an important consideration in the disclosure of information gained from a valuation, since smoothing is a one-way process. Armed with an unsmoothed series of results, it is possible for the recipient to create a smoothed series, but the reverse does not necessarily apply.

5. Financial Economics

5.1 *Market Price*

5.1.1 Financial economics is the study of the market pricing of Stock Exchange securities and other financial instruments. It is a body of theories which seek to explain the market price of financial instruments, the interrelationships between market prices and the development of market prices over time. The subject has mostly developed during the last 30 years; it underpins the modern business approach to shareholder value and the explosive development of the derivatives markets.

5.1.2 Only a few years ago the legislative framework of U.K. pension funds was such that market pricing of pension fund assets was of relatively minor importance. Trustees, sponsoring company and Scheme Actuary were all looking primarily at funding issues, particularly the setting and reviewing of contribution rates. Market price fluctuations were not regarded as important, except for the (usually undemanding) solvency checks. Now, the framework has changed and the market value of the assets is a much more important consideration, via annuity buyout costs, MFR, etc. It has, therefore, become more relevant to understand the principles of market pricing, insofar as these may affect the liabilities as well as the assets.

5.1.3 The relevance of market price, as distinct from any other assessment of value, is that it provides a common agreed measurement of value irrespective of the views and positions of market participants. In principle, the market price of a traded asset is settled on the basis of the maximum information about that asset which is available to all market participants.

5.1.4 Market pricing of financial assets has continued to develop in both breadth and depth. We now have index-linked bonds, gilt strips and highly liquid markets in stock options and futures, none of which existed 30 years ago.

5.1.5 Like any other body of science, the theories of financial economics are an attempt to explain relevant aspects of what we observe in the real world. As with any theory, the starting point is a collection of basic axioms which are deemed to be self-evident and true. The theory is deduced by logical reasoning on the basis of the stated axioms, and the theory is then tested against real life observations. If the fit is good, the theory is good. If the fit between theory and reality is less good, sometimes it is necessary to go back to fundamentals and adjust one or more of the axioms. The axioms of financial economics have been exposed to scrutiny from academics and practitioners over the last thirty years, and the basic principles have gained wide, though not universal, acceptance.

5.2 *The No Arbitrage Principle*

5.2.1 To illustrate the economic foundations, let us hypothecate that those people determining prices through their trading activities are financially rational and wish not to give up or to lose money in the absence of any compensating advantage to themselves. Assume, also, that these market participants are competing for the same stock of assets at any point in time. Then nobody is going to be able to get something for no risk or effort, because that can only be achieved at someone else's expense. This is the principle of 'no free lunch' or, in financial parlance, the principle of 'no arbitrage'. This principle can immediately be applied to the valuation of a pension, the future instalments of which can be exactly cash flow matched by an appropriate bond portfolio. According to the principle of no arbitrage, the market value of the pension must be exactly the same as the market value of the matching bond portfolio. To see this in elementary steps, let the market value of the asset portfolio be denoted by A and the corresponding value of the pension liability be L. If both the asset and the liability are tradeable, then we can apply the following reasoning.

5.2.2 Suppose first that L is greater than A. Any participant in the market can then acquire the liability, for which they will be paid a cash sum L, and can acquire the matching portfolio, for which they must pay the market price A. The cash flows from the asset portfolio will exactly match the payments due on the pension, so the net financial position of the participant is unaffected, apart from making a guaranteed 'free lunch' gain of L minus A. This is contrary to our axiom that other market participants are willing to lose money in order to allow this free lunch, so it cannot be true that L is greater than A.

5.2.3 Conversely, suppose that L is less than A. Consider an institution which has the liability to pay this pension, and suppose that, within its total asset portfolio, it arranges that a sub-portfolio with a value A is invested in the matching bond portfolio. Then the financial position of the institution is unchanged if it buys out the pension at price *L* and sells the asset at price *A*, and yet it gains a 'free lunch' profit of *A* minus *L* from the deal. Consequently *L* cannot be less than *A* either. The only logical conclusion is that *L* and *A* must be equal.

5.2.4 The principle of no arbitrage is the most powerful principle of financial economics. The simple and inescapable logic of the argument means that, even if the bonds are considered by some to be overpriced by reference to a subjective long-term criterion of value, then the market price of the corresponding pension must be correspondingly high. There is no room for subjective judgement where market pricing holds sway.

5.3 *Valuation of Liabilities*

5.3.1 Inspection of finance theory textbooks would suggest that finance theory is mostly concerned with market valuation of assets, not liabilities, but the emphasis in the textbooks on asset pricing is merely a reflection of the fact that many assets are widely traded, whilst most liabilities, other than financial instruments, are not. It should not be concluded that finance theory has little or nothing to say about liability valuation. The same principles which apply to asset pricing can be applied to liabilities as well, but with some differences to bear in mind.

5.3.2 We have already noted the fundamental principle of no arbitrage, and have shown how this assigns a market price to liabilities which can be cash flow matched by a portfolio of assets. Exley, Mehta & Smith (1997) pointed out that this principle can be extended to further classes of liability by using the principle of dynamic hedging. This is the principle which has been successfully applied in option pricing (via the Black Scholes option pricing formula), and which is based on the idea that the matching (or hedging) portfolio is varied in time, so that, at any instant of time, the asset/liability net position is immunised against changes in financial conditions during an arbitrarily short time interval. However, any such attempt to price liabilities is dependent on the model which is used for the pattern of short-term financial behaviour of the markets. The Black Scholes formula has been modified for practical use in option pricing over the years, because short-term stock market price movements do not behave exactly like the investment model which underlies the original formula. It is, therefore, necessary to design models which are appropriate to value liabilities, to test the predictions of the models against observed market prices (where such prices can be observed in a market) and to refine the underlying model as appropriate. For example, in the case of final salary pensions, the simplest model for future salary increases is price inflation plus a fixed margin of, say, 1.5% p.a. This would enable a suitably chosen index-linked bond portfolio to provide reasonably good cash flow matching to expected future salary increases, thus enabling the no-arbitrage principle to assign a market price to those liabilities; but is it reasonable to assume a fixed margin over price inflation, or should the model be refined? What happens to the margin when GDP growth accelerates or declines, or when interest rates rise or fall? More sophisticated models can be devised to tackle such questions; an example of this, in relation to future salary increases, is given by Smith (1998).

5.4 *Liability Risk*

5.4.1 Another principle of financial economics is based on the distinction between systematic and diversifiable risk. The textbooks usually explain the difference in terms of share prices, but we can do so in terms of mortality. In relation to any single person, there is usually much uncertainty about the year in which that person will die; but, to a large extent, the risk is diversifiable, because the uncertainty can be greatly reduced by looking at a large cohort of individuals in similar circumstances of age, sex, etc. That fundamental principle of life assurance is equivalent to the more general principle of financial economics, that the price to be associated with diversifiable risk is nil. This is easily demonstrated in the case of mortality. Suppose we are looking at £20,000 whole life assurance contracts for males aged 40. Suppose that a fair market price for a pure single premium payment to meet the liability is £3,300, and that this includes £300 for the price of the mortality risk. Then, leaving aside the separate matter of the costs of doing insurance business, an insurer can gather together a large cohort of similar contracts for 40-year-old males, can charge the pure net premium of £3,300 and can then pocket the 'free lunch' profit of £300 per policy, on the grounds that the mortality risk mostly disappears for a large enough group of policies. This is a breach of the no-arbitrage principle, which shows that there cannot be a material price attaching to the diversifiable mortality risk.

5.4.2 On the other hand, there is a systematic risk in mortality as well, because nobody knows whether, in future years, people will live shorter or longer lives than are predicted on the basis of current observed mortality rates. This longevity risk is the systematic or non-diversifiable risk, for which there is a price to be paid. In financial economics asset prices are effectively marked down by the price of systematic risk within the asset. The parallel for liability valuation is that values must be marked up by the price of systematic risk.

5.4.3 It is often observed that the cost of buying out pensions with insured annuities is more expensive than the reserving basis which is used by pension Scheme Actuaries. Part of the difference can be traced to differences in mortality rates, where insurance companies typically make a larger allowance for future mortality improvement than is normal in actuarial valuations. At least part of the difference in mortality assumptions may be attributable to an appropriate allowance being made by the insurance company for systematic longevity risk.

5.4.4 Another potentially important area of uncertainty is credit risk — or the degree of security attaching to a pension promise. The issue here (which comes up in discussion of reviewing the MFR) is whether a promise of a company pension should be regarded as providing the same degree of security as an insured pension, or whether a lesser degree of security is appropriate, and if so how much. In financial economics credit risk has a price, and so the value to be assigned to a pension must depend on the degree of security which is assigned to it. Thus, the degree of security needs to be defined before it is possible to place an economic value on the pension.

5.5 *Actuarial Values versus Market Prices*

5.5.1 The traditional actuarial valuation is based on a single deterministic set of assumption parameters which are designed to apply both to asset and liability valuations. Projected cash flows are discounted at a suitably chosen discount rate. In the traditional presentation of a pension fund valuation, assets do not appear at market value and the liability value is not adjusted to market either.

5.5.2 In complete contrast to this, the market value of a portfolio of assets is the result of the balance between supply and demand for the relevant assets at a point in time. Whilst individual market participants may make their own assumptions and judgements about the assets, the market price represents a 'democratic financial decision', which, actually, is not driven by any particular set of assumptions at all. Of course, it is usually possible to derive a model and assumptions which justify that market price, which leads to language about the market taking a view on something. Whether the market really does have a view is, perhaps, a matter of opinion and semantics.

5.5.3 In principle, a pure application of financial economics to the valuation of liabilities would lead to a 'market price' of the liabilities which is similarly derived from market price information, without any individual or subjective assumptions or judgements. In practice, matters are not so clear-cut, because of the nature of typical U.K. pension schemes. There are aspects of systematic risk within pension schemes which cannot be priced from the market very well, if at all. Examples of this are the extent to which future real salary growth net of price inflation will vary according to changing economic conditions, and demographic factors such as unknown future rates of withdrawal from service, early retirement, etc. The market does not readily supply prices for all these risk factors.

5.5.4 In the absence of a true traded market in final salary pensions, or in their various risk factors, it cannot be said that there is a uniquely correct market price to be associated with any given pension liability. Instead, financial economics offers a methodology for establishing an economic valuation, or a 'market consistent' valuation, as termed by Gordon (1999). This denotes a valuation which is consistent with the feasible range of market prices, if a true market were actually to exist.

5.5.5 The concept of a price range is entirely normal within asset pricing generally. The same concept must be applied to some types of liability. In the absence of trading, and without any matching assets, there is scope for judgement to be applied in certain liability valuation models and parameters within the market consistent framework. This notion is consistent with the degree of uncertainty that exists in some of the parameters underlying the pension promise.

5.6 *Summary*

In conclusion, financial economics is a body of theoretical analysis with application to the valuation of pension liabilities. In the absence of a market in pension liabilities, the application of financial economics principles leads not to a uniquely correct market price, but to a range of reasonable possibilities. However, the opportunity is provided for a more explicit and transparent approach to setting assumptions and applying judgement when required.

6. VALUATION METHODS CONSIDERED

6 .1 *Introduction*

6.1.1 The Working Party took the view that valuation methods which take assets at market value can be broadly grouped under four headings, although there could be variants within each one. In this section we describe each of these methods, numbering them 1 to 4. We then move on to analyse their properties in Sections 8 and 9. Appendix A sets out the methods algebraically, together with a practical example of the application of each method using market information as at 31 December 1998.

6.1.2 In each case we have described how the economic elements of the valuation basis are determined. These are used to calculate the past service value of liabilities, which is compared with the market value of assets. There is also the question of the future contribution rate. Throughout this paper we have adopted the approach that the future contribution rate is calculated using the same assumptions as the past service value of liabilities, giving a contribution rate in market value terms. It would be possible to adopt a hybrid approach of employing different methods for past and future service.

6.1.3 For comparison, we have also analysed the behaviour of what we call the traditional approach to an actuarial valuation, which we have termed Method 0.

6.2 *Method 0 (Traditional Method)*

6.2.1 This values both assets and liabilities using a discounted cash flow approach. The assessed value of assets represents the discounted present value of the expected income and capital proceeds from the scheme’s assets, usually expressed in the form of a market value adjustment (MVA) to those assets.

6.2.2 The MVA can be based on the proportions of assets actually held in each asset class, or alternatively based on a notional distribution of assets. Furthermore, it is common for most asset classes to be notionally invested in a representative index (e.g. U.K. equities valued as if invested in the FTSE Actuaries All-Share Index).

6.3 *Method 1 (Market Value Adjustment (MVA) Approach)*

6.3.1 This method is the most closely related to the traditional discounted cash flow approach, and provides a common way of arriving at individual or bulk transfer payments, for which a market value of liabilities is required at a relevant date.

6.3.2 The method takes the MVA (traditionally applied to the assets), and applies the inverse to the discounted value of the liabilities to give a market-adjusted value.

6.3.3 In this paper we have examined what we term Method 1, where the MVA is based on the actual proportion held in each asset class, and Method 1a, where the MVA is based on a notional portfolio which is intended to match the liability profile to some extent.

6.4 *Method 2 (Asset-Based Discount Rate)*

6.4.1 Under this method the market reference is made directly via the discount rate. We first derive an implied market discount rate for each asset class. For example, for gilt investments this is simply the gross redemption yield. For equity investments this involves determining the discount rate implied by the current market price and expected dividend and/or sale proceeds.

6.4.2 The overall valuation discount rate is then determined as a weighted average of these individual discount rates, based on the proportions invested in each asset class.

6.4.3 Different investment portfolios can be used to derive the discount rate applicable to the liabilities. Thus, this method could be based on the actual investment portfolio (say Method 2) or on a notional portfolio intended to match the liabilities (say Method 2a).

6.5 *Method 3 (Economic Valuation using Bond Yields)*

6.5.1 This is the method derived from financial economics. The inflation rate, discount rate and related assumptions are derived directly from market information.

6.5.2 At its simplest, the discount rate is taken as the gross redemption yield on a portfolio of conventional gilts with appropriate duration and convexity. The market inflation rate is derived by taking the difference between the yields on suitable portfolios of fixed-interest and index-linked gilts. The discount rate so derived is then used to value the liabilities.

6.5.3 The method described above makes use of a portfolio of assets, which leaves the minimum amount of risk with the fund sponsor as the liabilities are run off. Risks for which there is no obvious matching asset include salary growth in excess of price inflation, interest rates for very-long-term liabilities and demographic risks. For current pensioners with fixed pension increases the portfolio is made up of a suitable range of conventional gilts which match, precisely, the expected future pension payments. The only non-hedgeable risk is then systematic mortality risk. At the other end of the spectrum we have active members. Here the minimum risk portfolio is less clear.

6.5.4 Greater sophistication can be achieved in an economic valuation by consideration of the following:

— replacing the constant interest rate assumption with rates which vary according to the term to each liability payment. (for example, see Feldman *et al.*, 1998);
— use of discount rates based on yields on corporate debt, which therefore makes allowance for credit risk;
— valuation of caps and collars on pension escalation (e.g. Smith, 1998); and
— valuation of salary increases (e.g. Smith, 1998).

6.6 *Method 4 (Bond Yields plus Risk Premium)*

6.6.1 This method starts with the Method 3 discount rate (based on bond yields), but then adjusts it to take account of returns expected from other asset classes (e.g. equities).

6.6.2 This is done by adjusting (usually increasing) the discount rate by the addition of either a constant or a variable risk premium.

6.6.3 If a constant risk premium is used, the properties of this method are the same as Method 3, except that, effectively, the funding target is (usually) lower.

6.6.4 The more common approach is to introduce a variable risk premium. In reality this would be derived by a combination of market information and actuarial judgement. However, actuarial judgement is impossible to model accurately. We have, therefore, derived a formula which, by adjusting the discount rate to take account of market conditions, aims to maximise short-term stability in funding levels. This gives us an indication of how discount rates might be set if the aim is to achieve this stability. The derivation of this formula is set out in Appendix B. It must be emphasised that this is essentially a smoothing method. It is not possible to say what an actuary would advise under particular future market conditions, so the formula is a simplification.

6.7 *Smoothing*

Any method may be modified through use of a smoothing mechanism, either smoothing asset and liability values themselves or, for example, the resulting contribution rate. There are numerous methods of smoothing, and we do not propose to review these here. With any method of smoothing, care needs to be taken to ensure that the assumptions used for valuing the liabilities are consistent with the value placed on assets.

6.8 *Other Issues*

6.8.1 It is common for U.K. pension schemes to have a higher weighting in equity investment than might be deemed the matching portfolio. The use of prudent assumptions for future returns on equities would introduce, in effect, an implicit mismatching reserve under any of Methods 0, 1, 2 or 4, and this has often been the case where these methods have been used. It would, of course, be possible to allow for a mismatching reserve explicitly in conjunction with Method 3.

6.8.2 Many schemes have benefits which are not always fully defined (typically if there is an element of employer's discretion). These uncertainties could be modelled directly into the calculation process, or allowed for by an adjustment to the discount rate. Given the wide range of possibilities, we have not addressed this point further in this paper.

6.8.3 We have not reviewed international methods, such as FAS87 or the German book reserve model, though the principles underlying the liability valuation in FAS87 could be regarded as Method 3.

6.8.4 The above methods are analysed further in Section 8.

7. Criteria for Comparing Valuation Methods

7.1 *Introduction*

7.1.1 We now have five valuation methods to compare, i.e. the traditional assessed value approach, referred to as Method 0, and four other approaches detailed in the previous section, which take assets at market value.

7.1.2 In order to compare these methods, it is necessary to construct a common set of criteria against which these methods may be judged. Rather than refer to them as criteria, however, the Working Party settled on the word 'properties'. This enabled the Working Party to refer to a particular method as either featuring, or not featuring, a specified property, without necessarily commenting on whether that property was a desirable or undesirable outcome.

7.1.3 These properties are explained individually in Section 7.3. The various purposes for which actuarial valuations are carried out were described in Section 2. Below we also cover the various types of users of those actuarial valuations.

7.2 *Users of Actuarial Valuations*

7.2.1 The following users have a legitimate interest in one or more of the above valuation types.

7.2.1.1 *Trustees*

In general their objective is to protect the members' interests. To this end, they certainly seek sufficiency of assets and will monitor the various measures of funding described in Section 2. In addition, they will be conscious of the MFR requirements governed by Opra, implying a floor to funding. On the other hand, a ceiling is imposed by the excessive surplus legislation controlled by the Inland Revenue. Finally, they will also be interested in the effect of transactions, whether transfer values or other special payments.

7.2.1.2 *Sponsoring employers*

Traditionally, their normal objective has been to ensure that the pension fund is adequately, but not excessively, funded. Generally, they wish to use capital for their business rather than 'park' assets in pension funds over which they do not have control. For this reason they will normally wish to minimise contribution inputs to the scheme. In some cases an additional objective would be to make sure that any bulk transfer of pension liabilities, agreed as part of a corporate transaction, is likely to be covered by the assets received. Finally, the employer's financial results will depend on the pension expenses determined by the accounting provisions.

7.2.1.3 *Members*

Historically, members have relied upon the trustees to safeguard their interests. In general, members who are not trustees will not be as well informed as the trustees in terms of monitoring particular measures. However, it may reasonably be assumed that the members' objectives are, or should be, the same as those of the trustees.

7.2.1.4 *Opra*

Since April 1997 Opra have been charged with ensuring compliance with the requirements introduced by the Pensions Act 1995, including the MFR. The MFR sets a 'line in the sand', below which the asset values should not fall, having regard to the liability profile of each scheme.

7.2.1.5 *Inland Revenue*

Via the Pension Schemes Office (PSO), the Inland Revenue monitors tax relief on scheme contributions and investment proceeds. In order that the provision of tax relief is not abused, an excessive surplus test is carried out as part of each triennial valuation. This test is carried out using a conservative set of assumptions (set out in regulations) in order to be confident that any surplus assets revealed really are surplus to requirements.

7.2.1.6 *Accounting bodies*

They will wish to ensure that company accounts contain a true and fair reflection of the cost of accruing pension liabilities in accordance with the relevant accounting standard (currently undergoing some revision in the U.K.).

7.2.1.7 *Investors (and related parties)*

Active and prospective shareholders will normally rely on the accountants to check that pension expenses have been reported fairly. Other related parties include sponsors (e.g. merchant banks) of corporate new issues and the Stock Exchange. The latter has been active in the area of reporting on directors' pension arrangements following the Greenbury Report.

7.3 *Properties used to compare Valuation Methods*

7.3.1 In this section we describe the properties, or criteria, used to compare the different valuation methods referred to in Section 6.

7.3.1.1 *Consistency*

The relevance of consistency has already been discussed in Section 3.4. Here we consider consistency at two levels. Firstly, we consider whether assets and liabilities are included in the balance sheet at the same ‘currency’: either market values or assessed values. At the second level, we consider whether past service values are consistent with future service contribution payments.

7.3.1.2 *Simplicity*

This refers to the simplicity of determining the valuation assumptions and performing the necessary calculations.

7.3.1.3 *Durability*

Durability represents the ability of a valuation method to withstand sudden ‘shocks’. Two examples of a sudden shock are changes to U.K. taxation policy and changes in the way shareholder value is rewarded.

7.3.1.4 *Objectivity*

This refers to the degree to which a valuation method requires subjective assumption setting from an actuary. We would expect regulators and investors to be keen on objectivity, since such a feature permits a fair comparison between different schemes or between companies’ financial results.

7.3.1.5 *Targeting security of defined benefit*

A valuation method features this property if it aims to meet the defined benefit, both in the event of scheme wind-up and in the ongoing state.

7.3.1.6 *Stability of values*

We look at stability at two levels: stability of past service funding levels; and stability of contribution rates. These are tested, using an example scheme, in Section 8.

7.3.1.7 *Applicability to other valuation purposes*

Under this heading we discuss the extent to which different valuation methods could be used for all of the valuation purposes described in Section 2.

7.3.1.8 *Potential for impact on current U.K. pension scheme investment policy*

Even in a post-MFR environment, U.K. pension scheme investment is still biased towards equities. A valuation method which features this property has the potential for shifting current U.K. pension scheme investment policy towards greater bond investment. Whether this is desirable or undesirable we leave for the reader to decide.

7.3.1.9 *Potential for impact on current U.K. pension scheme funding policy*

Likewise, some valuation methods hold potential impact for changing the pace at which U.K. pension schemes are currently funded. Again, we do not comment on the desirability or otherwise, though, clearly, different interest groups would take different views on this.

8. NUMERICAL TESTING OF METHODS

8.1 *Introduction*

8.1.1 In this section we apply the various valuation methods described in Section 6, first using past data from 1985 to the end of 1998 (back testing), and second using simulated data generated by stochastic investment models proposed by Wilkie (1995) and Cairns (1999) (forward testing).

8.1.2 The purpose of these studies is to investigate how each method performs over time for a typical (but simplified) pension scheme. Thus, we look at the stability and mean of the funding level and of the contribution rate over time under each proposed method, and compare the results with the traditional discounted cash flow method.

8.1.3 Back testing will give us some comfort (or otherwise) that a proposed method would have given sensible answers in the past. The period for back testing has been chosen as that in which there have been sufficient quantities of index-linked gilts in issue for meaningful statistics to be available. Forward testing allows us to investigate how the methods perform in a much wider range of scenarios, enabling us to check for problems which could arise in the future, but which have not happened in the recent past.

8.1.4 In this section, for both back and forward testing, we have considered a simple pension scheme which has a stable membership distribution. The benefits provided are a single life pension from age 60, with pension increases in line with full price inflation. Full details of the scheme are given in Appendix C.

8.2 *Back Testing*

8.2.1 In this section the notional scheme has been modelled over the period from 31 December 1985 to 31 December 1998. Assets have been projected using actual returns on relevant indices over the period, with allowance for any income not used to pay benefits to be reinvested on a monthly basis. Assets were rebalanced to a particular target portfolio on a monthly basis.

8.2.2 The pension scheme was assumed to have reached the point where it has a stable membership distribution by 31 December 1985. From that point on, the liabilities were projected based on actual price and salary inflation. Salary increases were assumed to be in line with National Average Earnings increases.

8.2.3 In each case the same initial market value of assets has been used to aid comparability. Thereafter, employer contributions were assumed to be made in accordance with the valuation method chosen, based on annual actuarial valuations. Any surplus or deficit was amortised by adjusting the employer's future contribution rates to amortise surplus as a level percentage of salaries over 12 years (which is roughly the future working lifetime of the active membership).

8.2.4 In the back testing we have considered the following funding methods, as defined in Section 6.

8.2.4.1 *Method 0 (traditional)*

This is the traditional discounted value approach, where the market-value adjustment (MVA) is applied to the assets. The MVA is based on the proportions of assets actually held (but with overseas equities notionally valued as U.K. equities). Using the notation (i_v, e_v, r_v, d_v) for the valuation nominal rate of interest, rate of salary growth, rate of price inflation and gross dividend yield, respectively, we have used the following valuation basis:

$$i_v = 0.08, e_v = 0.06, r_v = 0.04, d_v = 0.04.$$

8.2.4.2 *Method 1 (MVA approach)*

This is as Method 0, but with the inverse of the MVA applied to the liabilities. The MVA is also applied to the contribution rate.

8.2.4.3 *Method 1a*

This is as Method 1, but with the MVA based on a notional portfolio which might traditionally be considered a closer match to the liabilities. As the liabilities are broadly evenly divided between active members and pensioners, this matching portfolio has been selected as 50% U.K. equities and 50% index-linked gilts.

8.2.4.4 *Method 2 (asset-based discount rate)*

The discount rate is set by reference to expected market returns on the asset classes held.

8.2.4.5 *Method 3 (economic valuation using bond yields)*

Here we define i_v as the yield on 15-year medium-coupon gilts (y_f), r_v is the difference between y_f and y_r, the real yield on 15-year index-linked gilts, and e_v is chosen such that the real salary assumption remains the same as above. d_v is not required.

8.2.4.6 *Method 4 (bond yields plus risk premium)*

As described in Section 6.6, for the purpose of modelling we have taken the approach of adding a variable risk premium to the discount rate derived for Method 3. The formula used to derive this premium is set out in Appendix B, and aims to maximise short-term stability in the funding level and contribution rate. It depends on the proportions invested in each asset class (long-dated fixed-interest, index-linked gilts and equities), and the durations of assets and liabilities. It represents a smoothing mechanism, and is not intended as a statement about the expected outperformance of equities relative to gilts. The constant term in the formula was chosen so as to give the same initial value of liabilities as at 31 December 1985 as Method 1.

8.2.5 In order to compare different methods, and in particular the volatility or otherwise, in their outcomes, we have determined certain summary statistics which are defined in Appendix D. Those calculated for the back testing are:

— *MF*1 — mean funding level;
— *MC* — mean contribution rate;
— *VF*1 — variance of funding level; and
— *VC*1 — variance of contribution rate.

8.2.6 However, a relatively smooth series of funding levels or contribution rates may demonstrate a high variance if they trend in a single direction over the entire period. Thus, we have also determined short-term volatility measures, which look at the average change year on year. These are:

— *VF*3 — short-term variance of funding level; and
— *VC*5 — short-term variance of contribution rate.

8.3 *Back Testing Experiments*

8.3.1 *Test 0: typical U.K. pension fund asset distribution*

8.3.1.1 This test assumed assets were invested 60% in U.K. equities, 20% in overseas equities, 5% in cash, 10% in fixed-interest gilts and 5% in index-linked gilts. This reflects a typical pension scheme investment portfolio over the period.

8.3.1.2 We have examined the funding position of our example scheme on the statutory bases currently in existence. This shows that neither the MFR (as set out in Actuarial Guidance Note 27, version 1.2), nor the statutory surplus test (as defined in the Income and Corporation Taxes Act 1988 and Statutory Instrument 1987/412) would have had any major impact on contribution rates in any of the tests.

8.3.1.3 The resulting funding levels and balancing contribution rates are shown in Figures 8.1 to 8.4. Summary statistics from this and all other tests are set out in Appendix E.

8.3.1.4 The main points to be noted from the figures, together with the statistics in Appendix E, are as follows:

— Funding levels under Methods 0 and 1 track, to a large degree, the behaviour of U.K. equity dividend payments over the period, with strong real dividend growth resulting in an improving funding position, and vice versa.
— In contrast, Method 3, in particular, tracks the behaviour of equity markets (in which the scheme is predominantly invested) against the index-linked gilt market (on which liability values are based).
— Methods 0 and 1 produce, as expected, very similar results. The difference between these methods is that under Method 1, both standard contribution rate and surplus to be amortised are subject to a market level adjustment, which they are not under the traditional approach (Method 0). However, Method 1 barely shows any increase in volatility of results from Method 0. The reason is primarily because there is a surplus, and the conditions under which surplus is ‘written up’ to market value also result in the standard contribution rate being written up, and the two effects cancel out to some

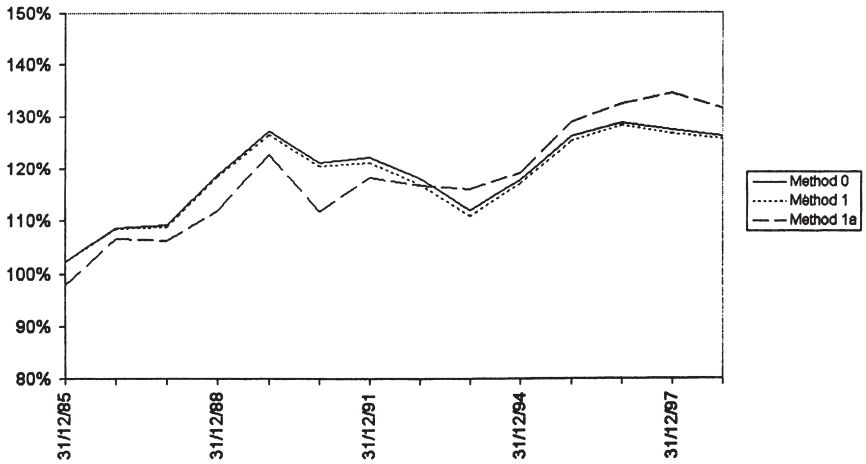

Figure 8.1. Funding levels; typical U.K. pension fund asset distribution; Methods 0, 1 and 1a

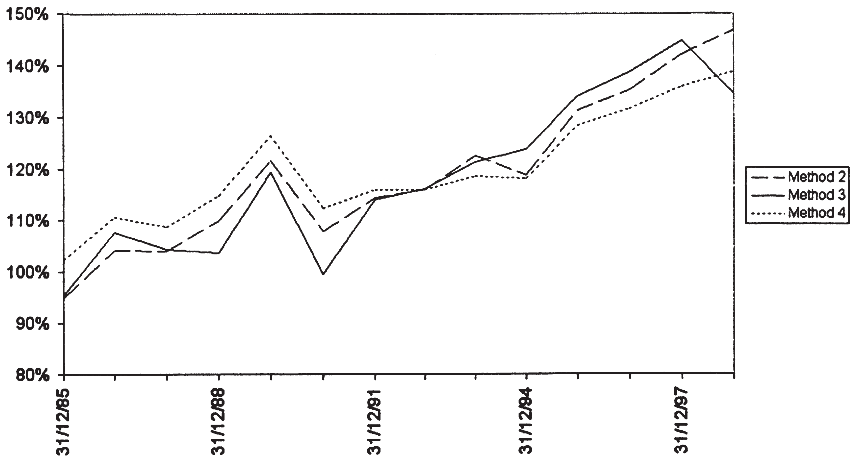

Figure 8.2. Funding levels; typical U.K. pension fund asset distribution; Methods 2, 3 and 4

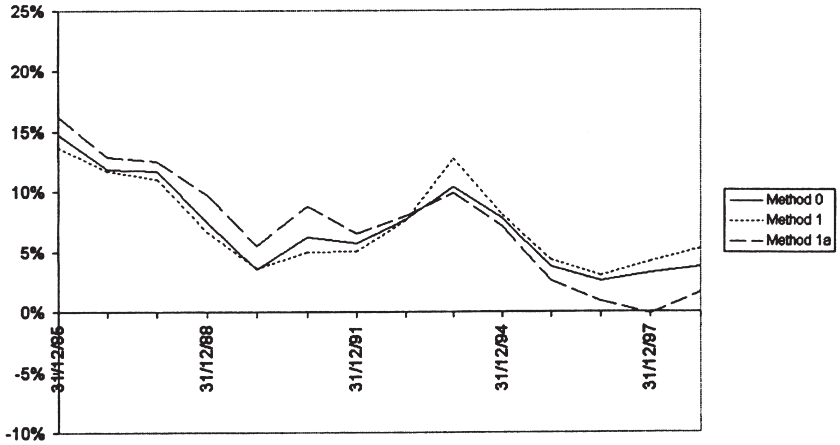

Figure 8.3. Contribution rates; typical U.K. pension fund asset distribution; Methods 0, 1 and 1a

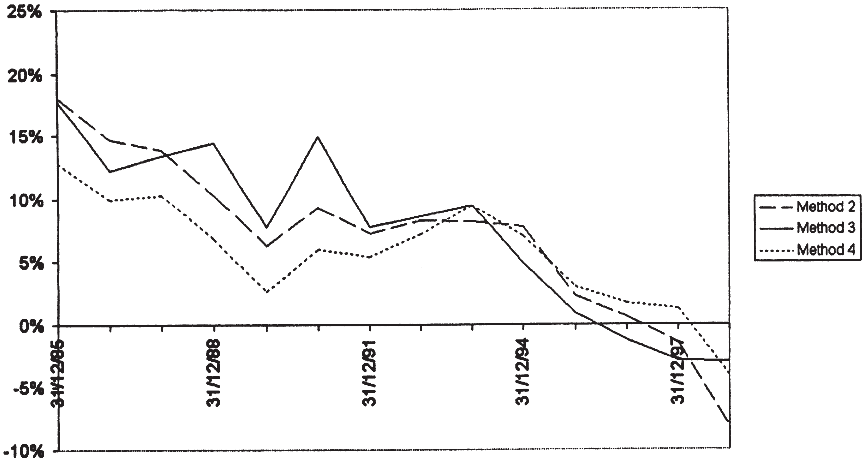

Figure 8.4. Contribution rates; typical U.K. pension fund asset distribution; Methods 2, 3 and 4

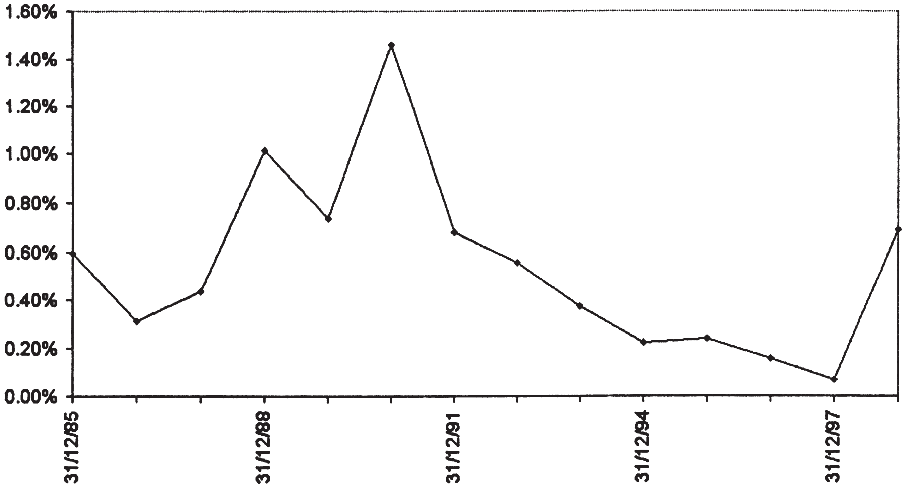

Figure 8.5. Risk premium under Method 4 in Test 0

degree. In the event of a deficit arising, the opposite would be true and volatility would increase.

— Once the notional portfolio moves further from the actual portfolio (Method 1a), the volatility of funding level increases.
— Method 2 appears more volatile than Methods 0 and 1, but less than Method 3.
— Method 3 exhibits the greatest volatility in results, as expected, due to the mismatch between assets and liabilities on this basis. This is particularly apparent in the late 1980s and early 1990s.
— Method 4 (by design) appears effective in reducing the volatility of results closer to that under Methods 0 and 1.
— Method 4 produces a risk premium varying between 0.07%p.a. and 1.46%p.a. on this asset distribution. The progression of the risk premium over time is shown in Figure 8.5.

8.3.2 *Test 1: effect of fixing the amortisation factor*

8.3.2.1 Previous studies (for example, Dufresne, 1988; Cairns & Parker, 1997) have indicated that varying the amortisation factor used to spread surpluses or deficits affects the stability of funding levels and contribution rates. To examine whether this is having an effect on the results of Test 0, we repeat Test 0, but using a constant factor of 0.1 for amortising surplus or deficit in the following year (that is, the employer's contribution is reduced by one tenth of the surplus). The results, which are only investigated for Methods 3 and 4, are set out in Appendix E.

8.3.2.2 The impact is that a marginal reduction in funding level volatility is achieved at the expense of greater volatility in contribution rates. Over the period in question, and on the assumptions used, this is due to a shorter average amortisation period with a fixed factor.

8.3.3 *Tests 2 and 3: effect of changing the asset distribution*

8.3.3.1 The substantial volatility, in particular of Method 3, is a result of the mismatch between assets, predominately invested in equities, and liabilities, denominated entirely in terms of index-linked gilt yields.

8.3.3.2 Tests 2 and 3 consider the impact of assuming a different asset distribution. They compare the results assuming investment in the 'typical' portfolio described in ¶8.3.1.1 with those assuming investment of 50/50 (U.K. equities/index-linked gilts) and 100% index-linked gilts respectively. The results for Methods 0, 1 and 3 are shown in Figures 8.6 to 8.11, and statistics are set out in Appendix E.

8.3.3.3 The shift towards index-linked gilts in the asset distribution serves to reduce volatility under all methods, but the reduction is clearly more marked in Method 3. Over the period examined, however, the outperformance of equities relative to index-linked gilts does produce dramatically differing mean contribution rates, with these being much higher as the asset distribution shifts towards index-linked gilts.

8.3.3.4 When the asset distribution is 100% index-linked gilts, there is least difference in behaviour between the various methods. Differences do remain, however, due to the imperfect match of assets and liabilities by term, and the 'currency' of Method 0 (i.e. assessed value rather than market value).

8.4 *Forward Testing*

8.4.1 Let us now consider how the different methods compare under a much longer, randomly-generated scenario. It was considered important to use more than one stochastic investment model. This reduces the risk that we make conclusions that are model dependent. The models we have used here are those of Wilkie (1995) and Cairns (1999), which will be referred to hereafter as Models 1 and 2 respectively. A comparison of these models is given in Appendix F. We have not considered here the effect of different parameter values in the two models, although this clearly is an important issue besides model variation.

8.4.2 In all, we show the results of a central experiment (experiment 0), and 9 others, in order to vary upper and lower funding level barriers, the amortisation factor for surpluses or deficits, and the asset distribution.

8.5 *Forward Testing Experiments*

8.5.1 The simulations used the same approach to determining the valuation assumptions for each method, as described in Section 8.2.

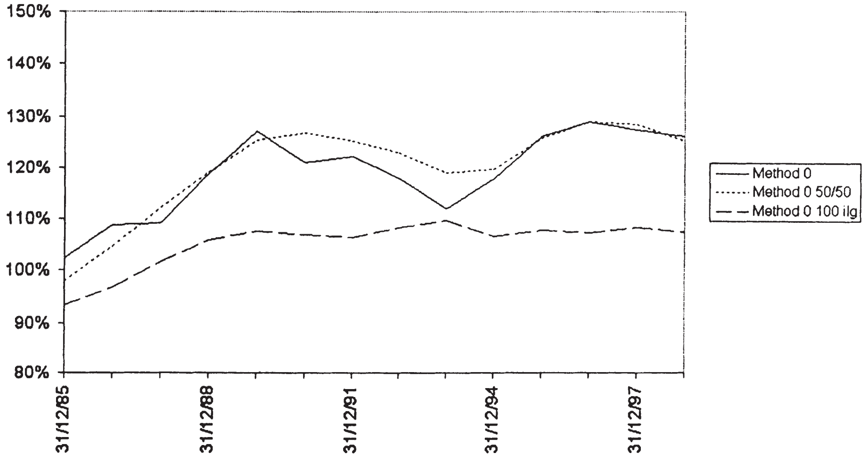

Figure 8.6. Funding level for Method 0 under different asset distributions

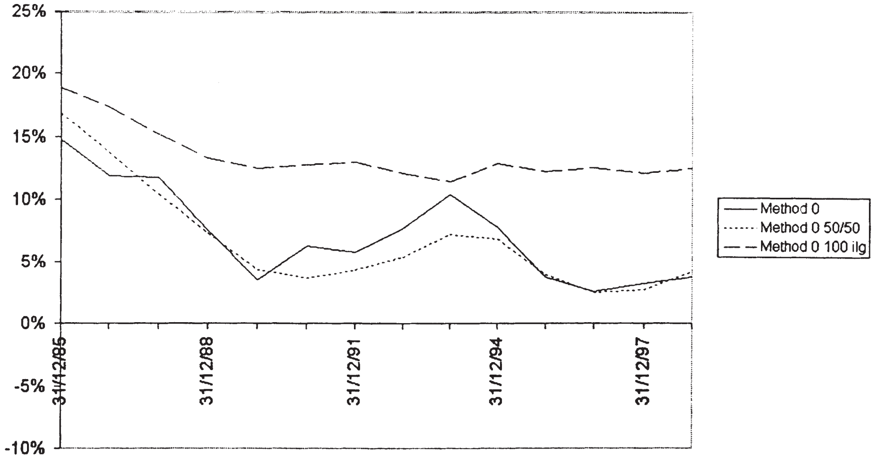

Figure 8.7. Contribution rate for Method 0 under different asset distributions

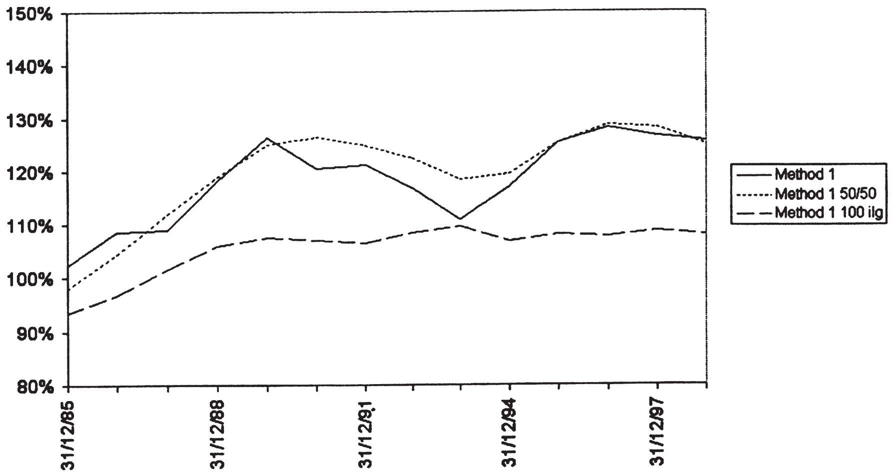

Figure 8.8. Funding level for Method 1 under different investment strategies

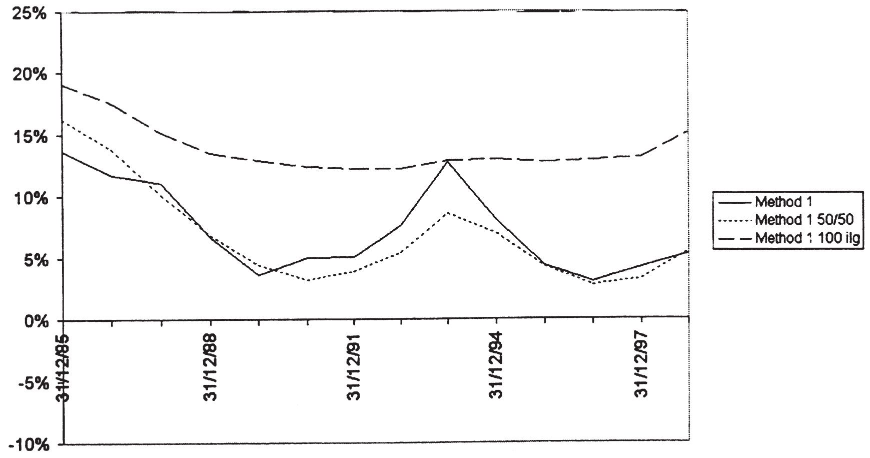

Figure 8.9. Contribution rate for Method 1 under different investment strategies

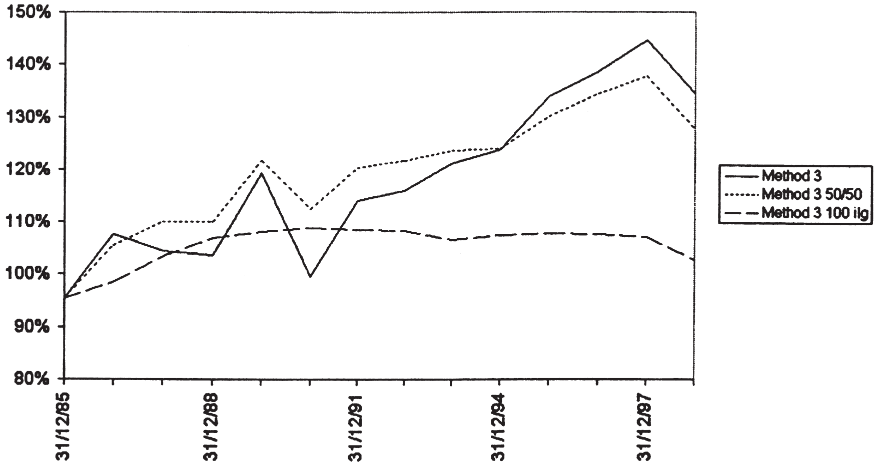

Figure 8.10. Funding level for Method 3 under different investment strategies

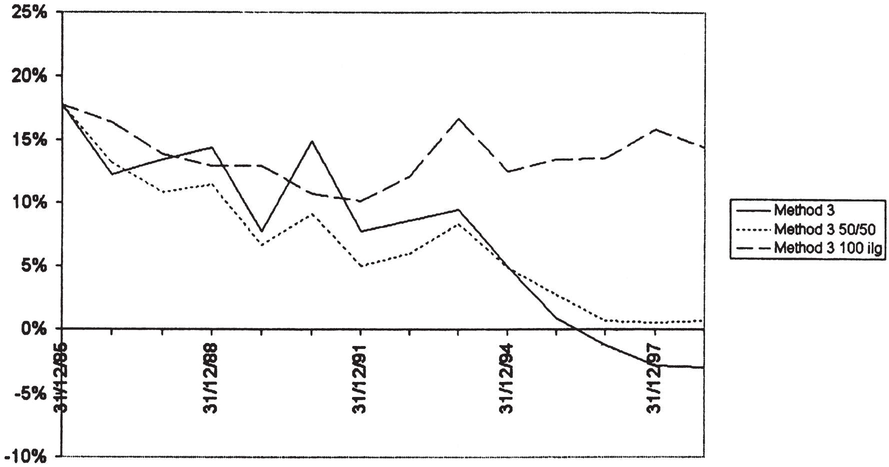

Figure 8.11. Contribution rate for Method 3 under different investment strategies

8.5.2 The asset distribution for the central experiment was taken to be 60% in U.K. equities, 5% in cash, 10% in fixed-interest gilts and 25% in index-linked gilts.

8.5.3 Upper and lower barriers were imposed as a simple means of mimicking a minimum funding requirement and statutory surplus regulations. Thus, any excess surplus over 25% of the liability on the regular valuation basis was required to be repaid immediately to the sponsor, while any deficit in excess of 25% of the liability was to be made up immediately.

8.5.4 The amortisation factor used in the central experiment was fixed at $k = 1/\ddot{a}_{\overline{m|}} = 0.1$ (this is higher than that using the average future working lifetime, but reflects the need to keep the funding level away from its boundaries). As referred to in Section 8.3.2, the reason for fixing k rather than fixing m is that the factor would vary as we change between valuation methods and valuation bases. Varying k, in addition to the valuation method, valuation basis and asset strategy, can cause some variation in the stability of funding levels and contribution rates (for example, see Dufresne, 1988; Cairns & Parker, 1997, and the comparisons in Experiments 1, 2 and 3). However, in later experiments we note that a change in the asset distribution can have a much more significant effect on stability than a change in k. Since we are aiming, here, to concentrate on the effect of the valuation method, it makes sense to remove this source of variation by fixing k.

8.5.5 In each experiment we considered the same 1000-year economic scenario generated by one of the two stochastic investment models (that is, Models 1 and 2). This ensured that differences between experiments using the same stochastic investment model could not be attributed to differences in sampling errors (differences between the two models will be subject to a small extent to sampling errors, since we are using two independent simulations).

8.5.6 A number of measures of stability are provided. For funding levels we give two basic values: $VF1$ is the long-term variance of the funding level; while $VF3$ is the short-term variance. A basic source of variability is the absolute size of the fund, that is, one fund, which is twice the size of another, will appear to be twice as volatile. Different funding methods can give rise to quite different fund sizes. Under such circumstances, comparison of absolute variances might give rise to misleading conclusions. Instead, we consider standardised variances $VF2$ and $VF4$, which remove the effect of fund size. Precise definitions of these measures can be found in Appendix D.

8.5.7 For contribution rates we give three principle measures of stability, depending upon the time horizon one wishes to consider. All are standardised to remove the effect of fund size. Measure $VC2$ gives the long-term variability of the contribution rate. Measure $VC3$ gives the average variance of the contribution over any 5-year period (perhaps a reasonable measure from the sponsor’s point of view). Measure $VC4$ gives the 1-year volatility in contribution rates (a measure of local smoothness). Measure $VC1$ is the long-term variance before standardisation.

8.5.8 Numerical results for the various experiments are given in Appendix G.

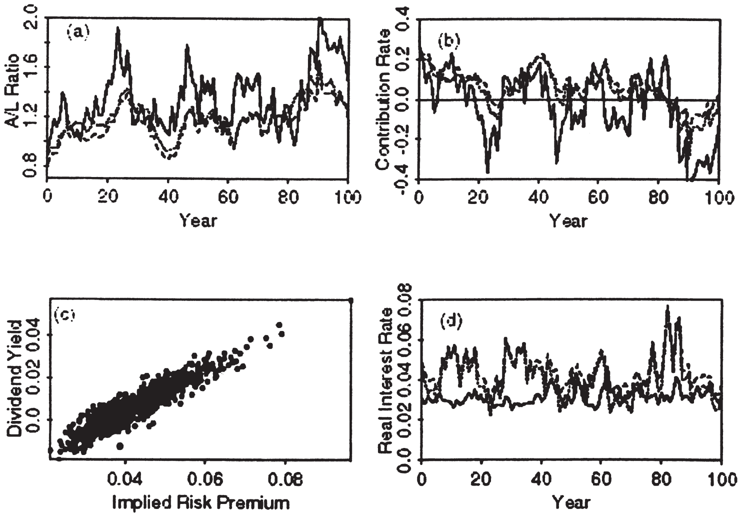

Figure 8.12. Results arising from a single, 100-year simulation using the Wilkie model; comparison of Methods 1, 3 and 4; in (a), (b) and (d): Method 3 – solid line; Method 1 – dotted line; Method 4 – dashed line; (a) variation of $A(t)/L(t)$ over 100 years; (b) variation of $CR(t)$; (c) dividend yield versus implied risk premium under Method 1. (d) variation of the real rates of interest

8.5.9 *Experiment 0*

8.5.9.1 This was the only experiment in which the barriers were set at 0.1 and 10 instead of 0.75 and 1.25. We consider results for both stochastic investment models.

8.5.9.2 Selected results are plotted in Figure 8.12. In Figure 8.12(a) we can see how the funding levels evolve over a 100-year period under Methods 1, 3 and 4. Clearly Methods 1 and 4 produce similar results, while Method 3 produces much more volatility. The latter observation is not entirely surprising, as it was not designed to produce stability. Contribution rates are plotted in Figure 8.12(b), with essentially the inverse of the patterns in Figure 8.12(a). Note that all methods produce regular periods of contribution refunds. This reflects the difference between average experienced real investment returns against the assumptions in the valuation basis. Figure 8.12(c) plots the dividend yield against the risk-premium implied within Method 1. The high degree of correlation led to the development of Method 4. Figure 8.12(d) plots the development of the valuation real rates of interest for the three methods.

8.5.9.3 Numerical results for this experiment are given in Tables W0 and C0 of Appendix G. We can make the following points:

— Methods 0, 1 and 4 produce similar levels of volatility over the short, medium and long terms. Method 3 produces much higher volatility in funding levels in the short term, but with similar levels of variability over longer periods to the other methods (comparing the statistics $VF4$ and $VF2$ respectively). Method 3 produces greater volatility in the contribution rate over all ranges.

— Although one-year unconditional means and variances in the two models are similar, the Cairns model produces returns which are more highly correlated from one year to the next. Cairns & Parker (1997) showed that this leads to higher variability in the funding level, and this is what we observe here: similar levels of short-term volatility and higher levels of long-term variance in Table C0.

— The strength of some valuation bases led, in some cases, to negative mean contribution rates. In reality, persistent surplus would probably result in benefit improvements such as discretionary pension increases rather than solely rebates to the sponsor.

— From Figure 8.12(d), we can see that the relative stability achieved under Methods 1 and 4 is achieved at the expense of rather volatile valuation rates of interest compared to Method 3.

8.5.10 *Experiment 1*

8.5.10.1 In this experiment we used the central assumptions described above. The only difference from Experiment 0 was the introduction of much more severe barriers at 0.75 and 1.25. These are intended to be reasonable approximations to the current minimum and maximum regulations in the U.K. The effect of the introduction of the narrower band can be seen graphically in Figure 8.13 (using a simulation generated by Model 1). Broadly the two funds progress in the same way over time, with deviations only when the funding level breaks through the upper barrier. The process rarely hits the lower barrier because of the high expected returns relative to the valuation basis.

8.5.10.2 Numerical results are detailed in Table W1.

8.5.10.3 Inevitably the funding level becomes more stable because of the constraints. Mean funding levels are lower, because the relative strength of the valuation basis means that the upper barrier comes into play much more frequently than the lower barrier. In contrast, the contribution rates become very much more variable. Primarily this is because of a small number of very large contribution refunds or deficit payments as a result of a breach of one of the barriers. Mean contribution rates are a little bit higher in this experiment because the mean funding level is lower. This means that there is less investment return to support contributions in the future.

8.5.10.4 This experiment also included a look at the effect of using a notional fund different from the actual structure of the portfolio (Method 1a). It can be seen from Table W1 that the notional fund results in a more stable funding level, but no obvious change in the stability of contributions. This is perhaps

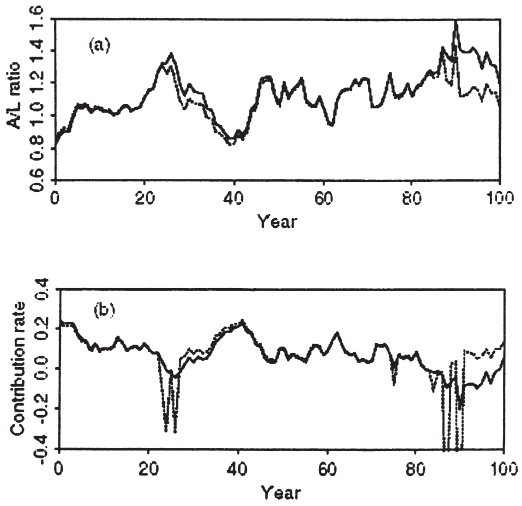

Figure 8.13. Method 4; comparison of (a) $A(t)/L(t)$ and $CR(t)$ with barriers at 0.1 and 10 (solid lines), or at 0.75 and 1.25 (dotted lines)

counterintuitive. However, we can note that Method 3 is an extreme version of the notional fund, as is the case where we assume a notional fund with 100% in U.K. equities, and both are significantly more volatile than the actual fund approach. We can infer from this that, as we work our way through the range of notional funds, there is a U-shape with a minimum variance close to, but not equal to, the actual fund.

8.5.10.5 We would stress that we are interested in qualitative results. Modelling more exactly the barriers and the way in which regulations require action if a barrier is breached could refine the model. However, this would not substantially alter the observations made below from a qualitative point of view.

8.5.11 *Experiments 1, 2, and 3*

8.5.11.1 We noted, in Experiment 1, that the barriers create additional variability. In these experiments we considered the effect of the amortisation rate k. If the barriers are considered to be a problem, then we should try to avoid hitting them. This means increasing k.

8.5.11.2 These three experiments took $k = 0.1$, 0.15 and 0.06 respectively. Numerical results are detailed in Tables W1, W2 and W3. A look at the variances shows that increasing k does, indeed, reduce volatility. This is because increasing k reduces the frequency at which the fund size breaches one of the barriers. However, this does not reveal the full picture. It is informative to look at the distribution of contribution rates. This is plotted in Figure 8.14. Where k is small the distribution is, in fact, quite closely packed around the mean, except for a fat

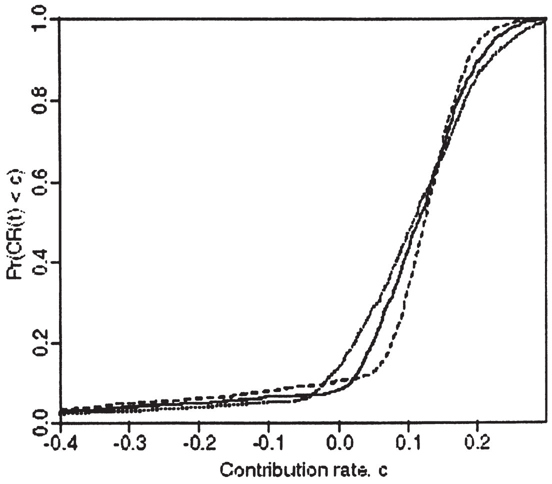

Figure 8.14. Experiments 1, 2 and 3; Method 4; cumulative distribution functions of the contribution rates for k = 0.15 (dotted line), k = 0.1 (solid line) and k = 0.06 (dashed line)

left-hand tail (caused by the number of refunds). This fat tail increases the variance noted in Table W3. However, a look at the shape of the distribution makes $k = 0.06$ look quite favourable. On the other hand, if we were to use a less conservative valuation basis, then we would be equally likely to hit either barrier. Under such circumstances, $k = 0.06$ would give rise to equally fat left and right-hand tails. In particular, the right hand represents additional contributions required under the MFR regulations, and may come at a bad time for the sponsoring employer.

8.5.12 *Experiments 4, 5 and 6*

8.5.12.1 Here we considered the effect of changing the investment strategy. The three experiments concentrate investments in equities and long-dated index-linked bonds in the ratio 80/20, 40/60 and 0/100 respectively. Numerical results, using stochastic investment Model 2, are given in Tables C4, C5 and C6. Funding levels and contribution rates for Experiments 4 and 6 are plotted in Figure 8.15.

8.5.12.2 From the tables and from Figures 8.15(a) and 8.15(b), we can see that switching into index-linked bonds has a very significant effect. This effect is much stronger than changing the amortisation factor k, noted in Experiments 1, 2 and 3. The residual variability where we are invested 100% in bonds is due both to the imperfect match between salaries and inflation and to the imperfect match from year to year between index-linked returns and inflation. The smoothing in Method 4 reduces the latter effect, and we can see this by comparing variances under Methods 3 and 4.

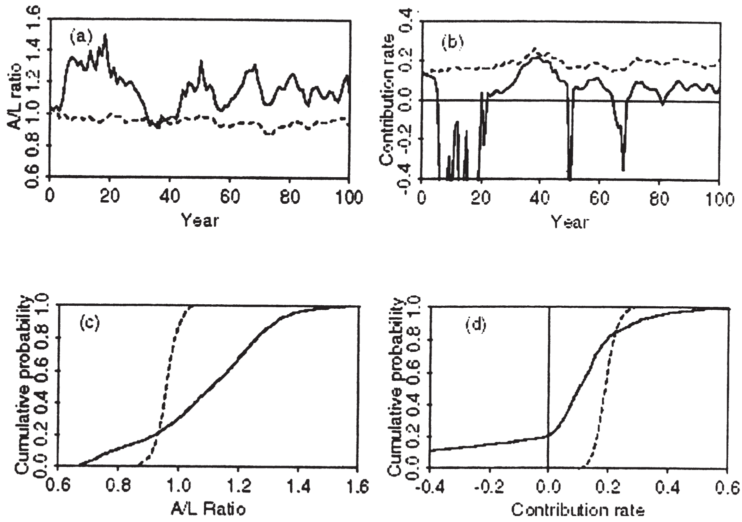

Figure 8.15. Method 4; dynamics of the fund under different investment strategies; (a) $A(t)/L(t)$; (b) $CR(t)$; (c) Cumulative distribution function of $A(t)/L(t)$; (d) cumulative distribution function of $CR(t)$; Experiment 4; 80% in equities and 20% in index-linked bonds (solid lines); Experiment 6; 100% index-linked bonds (dashed lines)

8.5.12.3 Over the one hundred years plotted, the equity strategy appears to perform rather better, with very few occasions when contribution rates are higher than the 100% bonds strategy. However, over the full 1000-year period (Figures 8.15(c) and 8.15(d)) we can see that the additional risks attached to equities do mean that there are a significant number of occasions (around 1 in 5) when the fund will appear to be in a much worse state, with higher contributions.

8.5.12.4 We can also see that mean contribution rates are higher as we put more into bonds. This is the counter balance to the lower variability. This is a reflection of two things. First, the average rate of return on the fund is lower. Second, the average fund size (amounts) is lower. This creates less investment return to offset contributions everything else being equal.

8.5.13 *Experiments 7, 8 and 9*

8.5.13.1 In Tables W7, W8 and W9 we give numerical results for three experiments in which we considered the effect of overseas equities using Model 1. We considered the following possibilities:

8.5.13.2 Experiment 7 has 20% in U.S. equities (used as a proxy for overseas equities), with the use of 1-year currency forwards to remove the exchange rate risk from 1-year returns.

8.5.13.3 Experiment 8 has 20% in U.S. equities, without the use of 1-year currency forwards.

8.5.13.4 In Experiment 9 the 20% is moved into U.K. equities.

8.5.13.5 It was assumed that overseas equities were treated as U.K. equities for the purpose of the valuations. It appears from the numerical results that the introduction of overseas equities marginally increases volatility under all methods, but this is more marked under Methods 0 and 1.

9. Results of Comparing Valuation Methods

9.1 *Introduction*

In this section we look at how the alternative valuation methods previously described compare against each of the properties introduced in Section 7.3, drawing, where necessary, on the simulated results from Section 8.

9.2 *Results of Comparison*

9.2.1 *Consistency*

9.2.1.1 All the methods investigated, including the traditional assessed value approach, show assets and liabilities in a consistent ‘currency’.

9.2.1.2 However, when we consider consistency between past service values and future service payments, the traditional assessed value approach does not feature this second aspect. This is because the contribution rate over a particular time period is calculated in assessed value terms, but paid at the prevailing market conditions without adjustment. Once contributions have been paid, the following actuarial valuation will apply a market value adjustment (MVA), thereby altering the assessed value attached to those contributions. This produces the peculiar capacity to create actuarial surplus or deficit, even if the inter-valuation experience has been exactly in line with the valuation assumptions. All of Methods 1 to 4 avoid this problem, because market conditions are taken into account when setting the contribution rate, though, of course, these market conditions may have changed by the time a particular contribution payment is actually paid.

9.2.2 *Simplicity*

9.2.2.1 Actuaries are used to performing assessed value calculations and setting the appropriate long-term valuation assumptions. Most actuaries would, therefore, regard the traditional approach as relatively straightforward to perform. Similarly, Methods 1 and 1a, which apply an MVA to liabilities determined using long-term assessed value assumptions, also feature this property.

9.2.2.2 We also regard Method 3 (economic valuation using bond yields) as simple to perform, since the appropriate yields are published daily in the *Financial Times.*

9.2.2.3 Method 2 (asset based discount rate) and Method 4 (bond yields plus risk premium), however, require an opinion on either current market expectations of equity dividend growth or the appropriate risk premium from other asset classes such as equities. At any one moment such views are likely to be highly contentious, and therefore we do not regard these methods as straightforward to operate.

9.2.3 *Durability*

9.2.3.1 The traditional assessed value approach focuses significantly on the dividend yield for the purpose of calculating the assessed value of assets. It is, therefore, exposed to 'shocks' that undermine the focus on dividends as the prime source of rewarding shareholders. We have seen two such shocks in recent years, firstly the taxation of U.K. company dividends to pension scheme investors, and secondly the growing use of share buy-backs and special dividends as an alternative means of rewarding shareholders. Given that these were prime reasons for the establishment of this Working Party, we conclude that the traditional approach no longer retains the property of durability.

9.2.3.2 Similarly, Methods 1 and 1a, which also rely on the dividend-based MVA approach, and Method 2, which uses expected dividend cash flows from the actual investment policy to determine a discount rate, fail to satisfy the durability test.

9.2.3.3 Method 3, however, is driven from bond yields at source. The durability of these methods is, therefore, dependent on the supply of government and corporate bonds. We expect that there will always be a need for government or for companies to borrow, hence the existence of a supply of debt is not really in doubt (though the adequacy of this supply may be called into question from time to time). Taxation is unlikely to be an issue, because any change will be immediately reflected in the redemption yield. Unlike equities, where companies can change habits to offset tax changes, there is no doubt as to the impact of tax on the investment return from bonds. We can think of no other possible shocks that would compromise the durability of Method 3.

9.2.3.4 The durability of Method 4 depends on the construction of the risk premium. A constant addition to bond yields is just as durable as Method 3, because the matching asset class is still bonds. On the other hand, a variable risk premium calculated using dividend yield data, such as the method shown in Appendix B, is exposed to uncertainty from the 'shocks' referred to in ¶9.2.3.1.

9.2.4 *Objectivity*

9.2.4.1 To the extent that any valuation method is prescribed by legislation (e.g. the current MFR), it can be considered objective. Here we focus on the objectivity, or otherwise, of non-prescribed valuations.

9.2.4.2 The traditional assessed value approach is not an objective valuation method, because the long-term investment return assumption is a subjective decision of the Scheme Actuary.

9.2.4.3 Likewise, Methods 1 and 2 are dependent on the actual investment portfolio of a scheme, and therefore cannot be considered as objective measures.

9.2.4.4 On the other hand, Method 3 (economic valuation using bond yields) is entirely objective, since it could be applied consistently across all pension schemes and avoids subjective judgement.

9.2.4.5 Method 1a (MVA approach with notional asset distribution) falls somewhere in the middle on the objectivity scale. It defines an asset distribution intended to match the liabilities in some way (although this still involves some subjectivity). It also involves a subjective assumption about implied dividend growth from equities. Method 4 (bond yields plus risk premium) again depends on the construction of the appropriate risk premium to add to bond yields. We have assessed these two valuation methods as 'mixed' for this property.

9.2.5 *Targeting security of defined benefit*

Method 3 (economic valuation using bond yields) most satisfies this property, since it targets the defined benefit in the event of scheme wind-up as well as at projected retirement age. All other methods target security of defined benefit at projected retirement age only.

9.2.6 *Stability of values*

9.2.6.1 Using the standardised volatility results from Appendix G (*VF*2 and *VF*4 — funding level, *VC*2, *VC*3 and *VC*4 — contribution rate), Methods 1 and 1a (MVA approach, using actual and notional asset distributions respectively) show volatility similar to the traditional method, the yardstick for measuring this property. It should be stressed that this assumes a typical U.K. pension fund asset distribution (i.e. with a heavy equity content).

9.2.6.2 Method 2 (asset based discount rate) is marginally more volatile, but still similar to the traditional approach.

9.2.6.3 Method 3 (economic valuation using bond yields) shows significantly greater volatility in funding level and contribution rate.

9.2.6.4 Method 4 (bond yields plus risk premium) shows stability of both funding levels and contribution rates. This is not surprising, since the construction of the risk premium in our example is specifically designed for this purpose. Indeed, any arbitrary smoothing rule can be applied separately to each of these methods.

9.2.7 *Applicability to other valuation purposes*

9.2.7.1 The force behind the International Accounting Standards Board's drive towards the use of market values for pension expense calculations means that the traditional assessed valuation approach is unlikely to be retained by the U.K. Accounting Standards Board. Thus, the traditional approach is unlikely to be applicable for all valuation purposes identified in Section 2.

9.2.7.2 Methods 1 and 2 would not be suitable for regulatory valuations, since the use of the actual investment policy would imply a discount rate that was scheme-specific.

9.2.7.3 We consider that the other methods could be used for all valuation purposes identified in Section 2.

Table 9.1. Comparison of alternative valuation methods

Property	Method 0 (traditional)	Method 1 (MVA, actual inv)	Method 1a (MVA, notional inv)	Method 2 (asset based d.r.)	Method 3 (economic – bond yields)	Method 4 (bond yields plus risk premium)
Consistency						
Same ‘currency’ for assets and liabilities	✔	✔	✔	✔	✔	✔
Treatment of past service values and contribution payments	x	✔	✔	✔	✔	✔
Simplicity of calculations	✔	✔	✔	x	✔	x
Durability e.g. to changes in taxation or reward of shareholder value	x	x	x	x	✔	–
Objectivity	x	x	–	x	✔	–
Targeting security of defined benefit	x	x	x	x	✔	x
Stability of values						
Past service funding levels	✔	✔	✔	✔	x	✔
Contribution rates	✔	✔	✔	✔	x	✔
Applicability to other valuation purposes	x	x	✔	x	✔	✔
Potential for impact on current U.K. pension scheme *investment* policy	x	x	✔	x	✔	✔
Potential for impact on current U.K. pension scheme *funding* policy	x	x	✔	x	✔	✔

Key: x = this method *does not* feature this property; – = mixed; ✔ = this method *does* feature this property

9.2.8 *Potential for impact on current U.K. pension scheme investment policy*

Method 1a (MVA approach using notional asset distribution), Method 3 (economic valuation using bond yields), and (depending on construction) Method 4 (bond yields plus risk premium) have the potential for altering U.K. pension scheme investment policy, principally the re-allocation of equities into bonds. Such re-allocation would inevitably arise as pension schemes moved to reduce mis-matching against the new liability benchmark. Method 3 is potentially very severe in this respect. A material reduction in expected investment returns, resulting from use of these methods, might be an extremely sensitive issue with many trustees and sponsoring employers.

9.2.9 *Potential for impact on current U.K. pension scheme funding policy*

9.2.9.1 The same methods referred to in ¶9.2.8 have the potential for altering U.K. pension scheme funding policy. Using Method 3 (economic valuation using bond yields) as an example, introducing a funding target of 100% of liabilities under this method would currently require a significant increase in contributions to most schemes, at least over the short to medium term, as sponsors try to rectify funding deficits against this target. Obviously, such a requirement would be very sensitive amongst scheme sponsors.

9.2.9.2 A possible solution to this problem would be to target a percentage of liabilities lower than 100%. Whether trustees and members could accept the psychology of targeting less than 100% funding is arguable. In addition, any government looking to use such an approach for a minimum funding standard will meet political objection if the new standard is interpreted, rightly or wrongly, as 'weak'.

9.3 *Summary of Results*

A summary of these results is shown in Table 9.1.

10. Conclusions

10.1 Actuarial science is a developing process. From the history described in the other sections, it can be seen how techniques change to reflect advances in thinking and technology. We have no reason to presume that this process will not continue into the future and believe that this review of valuation methods is simply another step along that road.

10.2 We have identified the following general points.

10.2.1 Depending on the purpose of the valuation, there is a wide range of techniques that can be adopted to calculate a value of liabilities to be compared with a set of assets taken at market value.

10.2.2 All of these methods allow for subjective input (both demographic and economic), to a greater or lesser extent, and so all can be called methods that allow for actuarial judgement, although some methods require less judgement than others.

10.2.3 It is not impossible, therefore, to arrive at similar (or even identical) liability calculations using different methods with appropriate actuarial judgement. It is also possible to have very different answers using some particular methods or differing judgement.

10.2.4 This shows us, once again, the power and professional responsibility that lies with actuarial judgement, and hence the requirement to apply this judgement correctly in terms of both the choice of method and any subjective assumptions used.

10.2.5 Inherent with this responsibility is a prerequisite to understand the purpose of the valuation and implications of the application of actuarial judgement. We therefore conclude that the profession should extend its education process to cover the understanding of methods of determining liabilities on a basis consistent with market values.

10.3 *Uses of the Valuation*

10.3.1 As a profession, we are unusual in that our advice is often used by several parties for several different purposes. The Working Party has recognised (as have many others in the profession) that often one ‘answer’ cannot suffice for several ‘questions’.

10.3.2 The Working Party believes that it is possible to classify the purposes of valuation calculations into those requiring no judgement (e.g. MFR), limited judgement (e.g. an accounting standard), or full judgement (e.g. setting contribution rates or sale or purchase calculations).

10.3.3 With such a classification, we believe that it is possible to identify methods which allow for greater or less control of the actuarial judgement referred to in ¶10.2.4.

10.4 *Observations*

10.4.1 Our conclusions from the specific testing we carried out were not altogether surprising. The adoption of a market value for assets must mean a volatile value for any comparable calculation of liabilities. This leads to volatile funding levels (unless assets and liabilities are closely ‘matched’) and volatile contribution requirements (unless long-term assumptions are used or smoothing is applied). It would appear to us that the holy grail of an objective methodology and smooth results is unattainable. Some compromise (or actuarial judgement) will still be required.

10.4.2 Our terms of reference requested us to assist in the MFR process now being undertaken by the profession. The Working Party’s conclusions are dependent on how the purpose of the MFR itself is defined. We believe that, if the current terms of reference are accepted, then a variation on Method 4 (bond yields plus risk premium) is appropriate. However, if the MFR is required to value near certainty of provision of the accrued benefit promise, only Method 3 (economic valuation using bond yields) would appear satisfactory.

10.4.3 Finally, it should be remembered that pension provision by employers is essentially a voluntary act. We would caution against dogma and overly prescriptive sets of actuarial assumptions and methods for setting contribution rates or as a basis for legislation. Not only does this potentially stifle the future application and development of more advanced techniques, but a prescriptive approach could also have wider economic and market implications, which may not serve the wider community.

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APPENDIX A

VALUATION METHODS CONSIDERED

A.1 In this appendix we set out the various methods algebraically, and practical examples of how assumptions could have been set as at 31 December 1998, for each of Methods 0 to 4.

A.1.1 At 31 December 1998, the key financial index figures were as follows:

FT-Actuaries All-Share Index gross dividend yield	2.92% p.a.
FT-Actuaries Fixed-Interest 15-year medium coupon yield	4.43% p.a.
FT-Actuaries over-5-year index-linked gilt yield (5% inflation)	1.94% p.a.

A.1.2 The long-term valuation assumptions, where used, are:

Investment return	8%	p.a.
Salary growth	6%	p.a.
Price inflation (pension increases)	4%	p.a.
Equity dividend growth	3.765%	p.a.

A.1.3 Combining the investment return and dividend growth assumptions above, using a traditional approach, gives a normal 'par' gross dividend yield on U.K. equities of:

$$\text{Ln}((1.08)/(1.03765)) = 0.04 \qquad \text{i.e. a 4\% par yield.}$$

A.2 *Method 0 (traditional)*

A.2.1 Algebraically the method is expressed as follows:

Liability cash flows	l_t at future time t
Discount factor	v based on the long-term rate of return on assets
Market value of assets	F
Asset model	A proportion P_i is invested in asset class i; this yields expected cash flows a_{it} at time t per unit of market value

Thus:

Value of liabilities	$L = \sum l_t v^t$
Market value adjustment (MVA)	$\sum\sum P_i \; a_{it} \; v^t$
Value of assets	$F \times MVA$

A.2.2 *Example*

A.2.2.1 Liabilities are valued using the long-term assumptions:

Investment return	8% p.a.
Salary growth	6% p.a.
Pension increases	4% p.a.

A.2.2.2 The equity MVA, MVA_e is calculated as:

$$MVA_e = 0.0292/0.04 = 0.730.$$

A.2.2.3 The fixed interest MVA, MVA_f is calculated as:

$$MVA_f = 0.0443 \times a^{(2)}_{\overline{15}|} + (1/1.08)^{15} = 0.702$$

where $a^{(2)}_{\overline{15}|}$ is calculated at 8% p.a. interest.

A.2.2.4 The index-linked MVA, MVA_r is calculated as:

$$MVA_r = 0.0194 \times a^{(2)}_{\overline{15}|} + (1.04/1.08)^{15} = 0.788$$

where $a^{(2)}_{\overline{15}|}$ is calculated at (1.08/1.04)−1 = 3.85% p.a. interest.

A.2.2.5 Based on an asset distribution of 80% equities, 10% fixed-interest, 5% index-linked and 5% cash, this gives:

$$MVA = 0.8 \times 0.730 + 0.10 \times 0.702 + 0.05 \times 0.788 + 0.05 \times 1 = 0.744.$$

A.3 *Method 1 (MVA approach using actual asset distribution)*

A.3.1 Algebraically the method is the same as Method 0, except that assets are taken at market value and liabilities are adjusted to a market value, as follows:

$$\text{Market adjusted liability value} = L/MVA.$$

A.3.2 *Example*

As at 31 December 1998, values of liabilities are determined using the long-term valuation assumptions, and then multiplied by 1/0.744.

A.4 *Method 1a (MVA approach using notional asset distribution)*

A.4.1 Algebraically the method is expressed as above.

A.4.2 *Example*

Assume a matching portfolio is taken to be 50% equities, 50% index-linked gilts, then the MVA as at 31 December 1998 would be:

$$MVA = 0.5 \times 0.730 + 0.5 \times 0.788 = 0.759.$$

A.5 *Method 2 (asset based discount rate)*

A.5.1 Algebraically the method is expressed as follows:

A.5.1.1 An appropriate asset model is used to derive an implied discount rate for each asset class i by solving the equation $\sum a_{it} v_i^t = 1$ for each i, since a_{it} are per unit of market value.

A.5.1.2 Under certain circumstances this can mean that the return on equities held for 15 years, say, is less then the return on a 15-year gilt.

A.5.1.3 A composite discount rate w is obtained by weighting the implied expected rates of asset return by asset model proportion $w^{-1} = \sum P_i v_i^{-1}$.

A.5.1.4 Alternatively, w can be determined by solving the equation $\sum w^t \sum P_i a_{it} = 1$.

A.5.1.5 The liability value is then obtained by discounting at w. $L = \sum l_t w^t$

A.5.2 *Example*

A.5.2.1 First determine the market perception of price inflation r, based on fixed-interest and index-linked gilt yields, as follows:

$$r = 1.0443/1.0194 - 1 = 0.0244 \qquad \text{i.e. } 2.44\% \text{ p.a.}$$

A.5.2.2 Determine the rate of dividend growth relative to this price inflation figure, using the real dividend growth assumption implicit in the long-term assumptions:

$$d_r = (1.03765/1.04) - 1 = 0.998 \qquad \text{i.e. } -0.2\% \text{ p.a.}$$

A.5.2.3 So, the dividend growth d is:

$$d = 1.0244 \times 0.998 - 1 = 0.0224 \qquad \text{i.e. } 2.24\% \text{ p.a.}$$

A.5.2.4 Then determine the return on equities i_e by solving the equation $\bar{a}_\infty \times 0.0292 = 1$, where $\bar{a}_\infty$ is calculated at a net rate of interest of $(1 + i_e)/1.0224 - 1$.

A.5.2.5 This gives $i_e = e^{0.0292} \times 1.0224 - 1 = 0.0527$ i.e. 5.27% p.a.

A.5.2.6 Returns on fixed interest and index-linked gilts are the relevant yields.

A.5.2.7 Based on a portfolio of 80% equity, 10% fixed-interest, 5% index-linked and 5% cash (say 5% p.a. expected return), the interest rate for valuation is:

$$i = 0.8 \times 0.0527 + 0.1 \times 0.0443 + 0.05 \times 0.0443 + 0.05 \times 0.05 = 0.0513$$
$$\text{i.e. } 5.13\% \text{ p.a.}$$

A.5.2.8 Pension increases are taken at market price inflation, namely 2.44% p.a.

A.5.2.9 Salary increases are taken at price inflation plus a margin, say 2%, giving 4.44% p.a.

A.6 *Method 3 (economic valuation using bond yields)*

A.6.1 Algebraically the liability valuation is the same as Method 0, except that the discount factor for the liabilities is taken from appropriate matching assets of appropriate term and the asset value is taken at market value.

A.6.2 *Example*

Assuming market pricing is taken from the gilt market, economic elements are taken directly from gilt market information, namely:

Investment return	4.43% p.a. (fixed-interest yield)
Salary growth	4.44% p.a. (market price inflation plus 2%)
Pension increases	2.44% p.a. (market price inflation).

A.7 *Method 4 (bond yields plus risk premium)*

A.7.1 Algebraically this method is identical to Method 3, except that the discount rate is adjusted according to (amongst other things) market conditions and investment policy.

A.7.2 *Example*

Assumptions are set as per Method 3. The discount rate is then adjusted by a risk premium. Using the approach adopted for the back and forward testing of Method 4 (the formula for which is set out in Appendix B), the addition to the discount rate as at 31 December 1998 is 0.69% p.a. giving:

Investment return	5.12% p.a. (fixed-interest yield)
Salary growth	4.44% p.a. (market price inflation plus 2%)
Pension increases	2.44% p.a. (market price inflation).

APPENDIX B

DERIVATION OF THE RISK PREMIUM EMPLOYED IN METHOD 4

B.1 In this appendix we justify the particular form of the adjustment given in Method 4 of Section 6. The key point is that, by suitable duration matching, we can justify Method 4 described below using a *model-free* argument. Thus, the method should sit equally well with any stochastic asset model.

B.2 In this appendix we give justification for why Method 4 is reasonably effective as a means of smoothing contribution rates. In particular, the method achieves its stability by duration matching. The argument is model free, and as such should sit equally well with any stochastic asset model.

B.3 *Notation*

y_f = yield on 15-year medium coupon fixed-interest gilts (*)
y_r = real yield on long-dated index-linked gilts (*)
d = gross dividend yield on equities
π = risk adjustment
= function of d, y_f and y_r
r = implied inflation
= $y_f - y_r$
δ = valuation force of interest
= $y_f + \pi$
M = current market statistics
= $\{d, y_f, y_r\}$.

(*) Yields are assumed to be continuously compounding rates.

B.4 The liability can be written as:

$$L = \sum_t c_t e^{rt} e^{-\delta t} = \sum_t c_t e^{-(y_r + \pi)t}$$

where c_t is the expected cash flow at t expressed in real terms relative to RPI (and includes allowance for the valuation real rate of salary growth over RPI).

B.5 Let the total assets be equal to A with a proportion p_i invested in assets i = 1 (equities), 2 (15-year 8% fixed-interest gilt) and 3 (long-dated 3.75% index-linked gilt).

B.6 Let $A_i = Ap_i$ be the amount held in asset i.

B.7 Suppose that the market conditions M change to:

$$M' = \{d', y_f', y_r'\} = \{d + \Delta d, y_f + \Delta y_f, y_r + \Delta y_r\}$$

where each of the changes, Δd, Δy_f and Δy_r are small. Then:

Equities	$A_1 \to A_1' = A_1\, d/d' \approx A_1(1-\tau_1\Delta d)$
Fixed interest	$A_2 \to A_2' = A_2\,(1-\tau_2\Delta y_f)$
Index-linked	$A_3 \to A_3' \approx A_3\,(1-\tau_3\Delta y_r)$

where $\tau_1 = 1/d$ is the equities duration, and τ_2 and τ_3 are the durations of the fixed-interest and index-linked assets.

Thus $A \to A' \approx A(1 - p_1\tau_1\Delta d - p_2\tau_2\Delta y_f - p_3\tau_3\Delta y_r)$.

Similarly $L \to L' \approx L\,(1-\tau_L\,(\Delta y_r + \Delta\pi))$, where τ_L is the duration of the liabilities and $\Delta\pi$ is the change we choose to make to π in response to changes in the other economic variables.

B.8 When we consider stability we are concerned (amongst other things) with the asset-liability ratio. Here:

$$\frac{A}{L} \to \frac{A'}{L'} \approx \frac{A}{L}(1 - p_1\tau_1\Delta d - p_2\tau_2\Delta y_f - p_3\tau_3\Delta y_r + \tau_L(\Delta y_r + \Delta\pi)).$$

B.9 For the greatest degree of short-term stability we therefore aim to have:

$$1 - p_1\tau_1\Delta d - p_2\tau_2\Delta y_f - p_3\tau_3\Delta y_r + \tau_L(\Delta y_r + \Delta\pi)) = 1$$

that is:

$$\Delta\pi = \frac{p_1\tau_1\Delta d + p_2\tau_2\Delta y_f + (p_3\tau_3 - \tau_L)\Delta y_r}{\tau_L}.$$

B.10 We should, therefore, define the equity smoothing adjustment (as a function of time) to be:

$$\pi(t) = \pi_0(t) + \frac{p_1\tau_1}{\tau_L}d(t) + \frac{p_2\tau_2}{\tau_L}y_f(t) + \frac{(p_3\tau_3 - \tau_L)}{\tau_L}y_r(t)$$

where $\pi_0(t)$ is a smooth function (possibly deterministic and possibly constant).

B.11 The same method can be applied to the contribution rate. This is more complex, since the interest rate adjustment needs to depend upon the factors above plus pensioners' liabilities, the asset-liability ratio and the current funding level.

B.12 *Example*

In practice, all of the durations used in the calculation above vary over time with economic conditions. In practice, we use approximations to the durations based upon central assumptions. Thus, in the simulations, we took the fixed values $\tau_1 = 25$, $\tau_2 = 12$, $\tau_3 = 15$ and $\tau_L = 20$, and each year calculate directly the equity smoothing adjustment:

$$\pi(t) = \pi_0 + \frac{p_1 \tau_1}{\tau_L} d(t) + \frac{p_2 \tau_2}{\tau_L} y_f(t) + \frac{(p_3 \tau_3 - \tau_L)}{\tau_L} y_r(t)$$

using the appropriate proportions in each asset class.

APPENDIX C

MODEL PENSION SCHEME

C.1 The model scheme used in the back testing and forward simulations is as follows.

C.1.1 Benefits are 1/60 of salary at the date of retirement for each year of service.

C.1.2 Pensions are payable annually in advance and increase in line with full RPI.

C.1.3 There are no other benefits.

C.1.4 There are no member contributions.

C.1.5 The demographic elements of the valuation basis used were as follows:

Mortality	Pre-retirement	Zero
	Post-retirement	PA(90) minus 2 years
Other decrements	None	
New entrants	For forward modelling 10 p.a. at each of ages 20 to 29 inclusive. For back testing, 4 p.a. at each of ages 20 to 39, 2 at ages 40 to 49 and nil above this age	
Salary structure	All members receive the same salary p.a.	
Salary increases	In line with national average earnings, determined using a stochastically generated salary index for forward simulations	

C.1.6 The membership is assumed to have been in a stable state for many years.

C.1.7 On the traditional valuation basis used in Section 8 (Method 0), actives make up approximately 50% of the liabilities on the above valuation basis.

C.1.8 Liabilities are always valued as past service benefits only, and make full allowance for future expected salary increases.

C.1.9 Valuations are conducted using the projected unit method.

APPENDIX D

NUMERICAL RESULTS OF SIMULATIONS

In the tables that follow, we use the following notation:

n = number of years in simulation
$F(t)$ = $100\,A(t)/L(t)$ = funding level at t
$CR(t)$ = $100C(t)/TSR(t)$ = percentage contribution rate
$F_e(t)$ = $A(t)/L_e(t)$ = funding level as a percentage of the Method 3 liability

$CB5(t)$ = $\frac{1}{5}(CR(t-2)+CR(t-1)+\cdots+CR(t+2))$ = 5-year sample mean

$SVC5(t)$ = $\frac{1}{5}\sum_{s=t-2}^{t+2}(CR(s)-CB5(t))^2$ = 5-year sample variance

$\Delta CR(t)$ = $CR(t)-CR(t-1)$
$\Delta F(t)$ = $F(t)-F(t-1)$

$MF1$ = $\frac{1}{n}\sum_t F(t)$ = mean funding level

$MF2$ = $\frac{1}{n}\sum_t F_e(t)$ = standardised mean

$VF1$ = $\frac{1}{n}\sum_t (F(t)-MF1)^2$ = long-term variance of $F(t)$

$VF2$ = $VF1\times 10000/MF2^2$ = standardised variance of $F(t)$

$VF3$ = $\frac{1}{n-1}\sum_t (\Delta F(t))^2$ = short-term volatility

$VF4$ = $VF3\times 10000/MF2^2$ = standardised short-term volatility

MC = $\frac{1}{n}\sum_t CR(t)$ = mean CR

$VC1$ = $\frac{1}{n}\sum_t (CR(t)-MC)^2$ = long-term variance of $CR(t)$

$VC2$ = $VC1\times 10000/MF2^2$ = standardised long-term variance

$VC3 = \left(\frac{1}{n-4} \sum_t SVC5(t) \right) \times 10000 / MF2^2 =$ standardised mean 5-year variance

$VC4 = \left(\frac{1}{n-1} \sum_t \{\Delta CR(t)\}^2 \right) \times 10000 / MF2^2 =$ standardised short-term volatility

$VC5 = \frac{1}{n-1} \sum_t (\Delta CR(t))^2 =$ short-term volatility

APPENDIX E

NUMERICAL RESULTS OF BACK TESTING

Test 0 Typical U.K. pension fund asset distribution

	$MF1$	$\sqrt{VF1}$	$\sqrt{VF3}$	MC	$\sqrt{VC1}$	$\sqrt{VC5}$
Method 0	118.95	8.21	5.61	7.15	3.81	2.57
Method 1	118.34	8.08	5.57	7.27	3.61	2.87
Method 1a	118.16	10.94	6.30	7.29	4.87	2.69
Method 2	119.20	15.14	7.75	6.91	6.84	3.25
Method 3	118.33	15.38	10.05	7.47	6.85	4.16
Method 4	119.84	10.73	6.82	5.65	4.42	2.88

Test 1 Effect of fixing amortisation factor

	$MF1$	$\sqrt{VF1}$	$\sqrt{VF3}$	MC	$\sqrt{VC1}$	$\sqrt{VC5}$
Method 3	117.86	14.91	10.04	6.72	7.59	4.45
Method 4	119.48	10.39	6.79	5.09	4.97	3.09

Test 2 Investment strategy 50% U.K. equities, 50% index-linked gilts

	$MF1$	$\sqrt{VF1}$	$\sqrt{VF3}$	MC	$\sqrt{VC1}$	$\sqrt{VC5}$
Method 0	120.05	9.24	4.57	6.65	4.27	2.10
Method 1	119.82	9.17	4.55	6.79	4.11	2.29
Method 3	119.62	11.75	6.65	6.96	5.14	2.76

Test 3 Investment strategy 100% index-linked gilts

	$MF1$	$\sqrt{VF1}$	$\sqrt{VF3}$	MC	$\sqrt{VC1}$	$\sqrt{VC5}$
Method 0	105.25	4.70	2.40	13.44	2.16	1.10
Method 1	105.50	4.81	2.38	13.91	2.10	1.12
Method 3	105.50	4.11	2.34	13.75	2.24	2.26

APPENDIX F

STOCHASTIC INVESTMENT MODELS

F.1 Here we give a brief comparison of the two stochastic investment models used in the simulations in Section 8. The first model considered was the Wilkie model. This is well documented in the paper by Wilkie (1995).

F.2 The second model has been developed by one member of the Working Party. Technical details of this paper can be found in the paper by Cairns (1999). The model can be considered in the form of a cascade model, although the model is Markov, meaning that all market variables in fact have equal status. Key features of this second model are:

— At the top layer the model considers nominal and real rates of interest.
— Retail price inflation is driven by the difference between short-term nominal and real rates of interest. This ensures that expected returns on index-linked bonds are consistent with the risk-free (nominal) rate of interest.
— All rates of interest evolve within an arbitrage-free framework (see, for example, Flesaker & Hughston, 1996; Rogers, 1997; Rutkowski, 1997).
— All nominal rates of interest are positive.
— Total returns on equities are driven by the risk-free rate of interest plus a risk premium plus a random error.
— Equity price changes are positively correlated with price changes on fixed-interest bonds.
— The model is constructed within a continuous-time framework, but is simple to operate (as it is here) in discrete time.
— The arbitrage-free nature of the model gives it a coherent short-term structure. In addition, the model is designed to give realistic long-term properties such as autoregression.
— The model incorporates factors which fluctuate in long cycles, so that, for example, prices can go through long periods of low, stable inflation and other long periods of high, unstable inflation. This feature produces results which can appear to be non-stationary over periods of, say, 100 years while, in fact, being genuinely stationary over much longer periods.

F.3 For the purpose of this paper, the Cairns model has been calibrated to give comparable real returns on each asset class when compared to the Wilkie model. Nominal returns in the Cairns model are on average 3% lower than the Wilkie model. This is partly dictated by the fixed-interest model under which long-term par yields must span the range 2.5% to 15% with reasonable probability. This calibration reduces the risk that differences between the two sets of results are the result of parameter risk rather than due to differences between the structures of the two models. Note that the existence of long cycles means that the model is entirely consistent with, say, the last 50 or 100 years of U.K. data, even though, in the long run, mean returns are quite different.

F.4 Selected statistics relating to the two models are given in Tables F.1 and F.2.

F.5 Note that the qualitative conclusions drawn from the results of forward testing using these two models are not particularly sensitive to the parameter values used. For example, our observations would not change substantially if equities yielded 1% more or 1% less than assumed in our calculations.

Table F.1. Sample mean real returns on different assets on the two models (that is, the mean of the $TRI(t) \times RPI(t-1)/TRI(t-1) \times RPI(t)$, where $TRI(t)$ is the total return index for the given index and $RPI(t)$ is the retail prices index, and not the mean of the log-returns); the Wilkie model uses irredeemable fixed-interest and index-linked bonds while the Cairns model uses 25-year par bonds; under the Wilkie model the cash account was constructed using a rolling portfolio in 1-year zero-coupon bonds; under the Cairns model a rolling portfolio in 1-month Treasury bills was used; U.S. equities were considered with and without the use of one-year currency forward contracts; in contrast to the Wilkie model, price inflation under the Cairns model is heavily skewed to the right, making periods of negative inflation (when nominal rates of interest are low) less severe than they would otherwise be

Class	Asset	Wilkie mean %	S.D. %	Cairns mean %	S.D. %
1	U.K. equities (net)	7.6	21.2	7.5	20.6
2	U.K. cash	2.2	4.7	2.5	1.4
3	U.K. consols (W)/ 25-year fixed-interest par (C)	4.3	12.3	4.0	11.4
4	U.K. 25-year index-linked par	3.7	12.2	3.6	4.1
5	U.S. equities (with currency forward)	8.3	26.0	–	–
6	U.S. equities (without currency forward)	9.4	28.6	–	–
Real salary growth		2.0	2.6	1.9	2.5
Annual price inflation		5.1	3.8	2.1	5.0

Table F.2. Sample correlation matrices for real returns on the Wilkie model(ρ_W) and on the Cairns model (ρ_C)

$$\rho_W = \begin{pmatrix} 1.00 & 0.21 & 0.26 & 0.05 & 0.61 & 0.54 \\ 0.21 & 1.00 & 0.54 & 0.09 & 0.18 & 0.05 \\ 0.26 & 0.54 & 1.00 & 0.40 & 0.19 & 0.11 \\ 0.05 & 0.09 & 0.40 & 1.00 & 0.01 & 0.00 \\ 0.61 & 0.18 & 0.19 & 0.01 & 1.00 & 0.87 \\ 0.54 & 0.05 & 0.11 & 0.00 & 0.87 & 1.00 \end{pmatrix}$$

$$\rho_C = \begin{pmatrix} 1.00 & 0.10 & 0.58 & 0.03 & - & - \\ 0.10 & 1.00 & 0.26 & 0.27 & - & - \\ 0.58 & 0.26 & 1.00 & 0.07 & - & - \\ 0.03 & 0.27 & 0.07 & 1.00 & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \end{pmatrix}$$

APPENDIX G

FORWARD SIMULATION RESULTS

G.1 In the tables below the quoted statistics are the sample means and variances calculated from a single 1000-year scenario generated by either the Wilkie (1995) or the Cairns (1999) stochastic investment model. Apart from the choice of model, we always use the same scenario in each set of calculations to eliminate differences due to sampling errors.

G.2 The various statistics are defined in Appendix D. *VF*1, *VF*2, *VC*1 and *VC*2 are all long-term variances; *VC*3 measures 5-year volatility; *VF*4 and *VC*4 measure short-term volatility.

Table W0. Experiment 0; Model W; central basis, but with barriers at 0.1 and 10

	*MF*1	*MF*2	√*VF*1	√*VF*2	√*VF*4	*MC*	√*VC*1	√*VC*2	√*VC*3	√*VC*4
Method 0	119.4	108.7	21.1	19.4	6.4	4.9	12.3	11.3	3.7	3.8
Method 1	119.0	108.4	20.8	19.2	6.5	5.0	12.6	11.6	4.1	4.3
Method 2	127.9	126.6	23.9	18.9	6.3	1.8	16.1	12.7	5.5	5.9
Method 3	132.4	132.4	29.5	22.3	13.2	–1.2	19.6	14.8	7.8	8.8
Method 4	113.5	97.0	20.2	20.8	7.5	7.3	11.1	11.4	4.0	4.2

Table C0. Experiment 0; Model C; central basis, but with barriers at 0.1 and 10

	*MF*1	*MF*2	√*VF*1	√*VF*2	√*VF*4	*MC*	√*VC*1	√*VC*2	√*VC*3	√*VC*4
Method 0	134.4	146.3	46.8	32.0	4.3	–4.0	27.6	18.9	3.3	2.6
Method 1	130.6	142.0	38.4	27.0	4.1	–3.1	25.8	18.2	4.1	3.6
Method 3	135.1	135.1	41.4	30.6	13.3	–2.4	26.6	19.7	8.5	8.6
Method 4	125.8	123.7	37.3	30.2	5.9	0.7	22.2	17.9	4.3	4.1

Table W1. Experiment 1 ; Model W; central basis, Method 1(a) uses a notional portfolio of 50% equities and 50% index-linked gilts; all other methods (other than Method 3) use the actual portfolio

	*MF*1	*MF*2	√*VF*1	√*VF*2	√*VF*4	*MC*	√*VC*1	√*VC*2	√*VC*3	√*VC*4
Method 0	110.2	100.3	12.9	12.9	6.8	6.5	18.8	18.8	14.6	19.3
Method 1	110.1	100.2	13.0	12.9	6.9	6.6	20.1	20.1	16.4	21.8
Method 1(a)	113.8	117.0	11.8	10.1	5.9	3.0	24.5	20.9	16.9	22.4
Method 2	113.6	112.4	12.4	11.1	6.9	4.8	25.2	22.4	18.8	24.8
Method 3	109.9	109.9	16.6	15.1	14.6	2.6	42.1	38.3	36.1	49.1
Method 4	106.6	91.1	13.7	15.0	8.0	8.2	18.7	20.6	16.8	22.4

Table W2. Experiment 2; Model W; central basis, but with an amortisation factor of $k = 0.15$

	$MF1$	$MF2$	$\sqrt{VF1}$	$\sqrt{VF2}$	$\sqrt{VF4}$	MC	$\sqrt{VC1}$	$\sqrt{VC2}$	$\sqrt{VC3}$	$\sqrt{VC4}$
Method 0	108.3	98.6	11.7	11.8	6.8	6.8	17.3	17.5	12.7	16.4
Method 1	108.2	98.5	11.7	11.9	6.8	6.9	18.3	18.6	14.1	18.5
Method 3	108.9	108.9	15.9	14.6	14.6	2.7	40.9	37.6	35.2	47.2
Method 4	105.4	90.1	12.0	13.3	8.0	8.4	17.3	19.2	14.8	19.6

Table W3. Experiment 3; Model W; central basis, but with an amortisation factor of $k = 0.06$

	$MF1$	$MF2$	$\sqrt{VF1}$	$\sqrt{VF2}$	$\sqrt{VF4}$	MC	$\sqrt{VC1}$	$\sqrt{VC2}$	$\sqrt{VC3}$	$\sqrt{VC4}$
Method 0	112.0	101.9	14.0	13.7	7.0	6.2	20.9	20.5	17.0	22.9
Method 1	111.8	101.7	14.1	13.9	7.1	6.3	22.2	21.8	18.7	25.2
Method 3	110.8	110.8	17.2	15.5	14.7	2.5	43.4	39.2	37.3	51.2
Method 4	107.6	92.0	15.2	16.5	8.1	8.0	20.1	21.9	18.6	25.1

Table C4. Experiment 4; Model C; central basis, but with 80% equities and 20% index-linked

	$MF1$	$MF2$	$\sqrt{VF1}$	$\sqrt{VF2}$	$\sqrt{VF4}$	MC	$\sqrt{VC1}$	$\sqrt{VC2}$	$\sqrt{VC3}$	$\sqrt{VC4}$
Method 0	112.7	127.0	19.5	15.4	3.9	–4.8	36.9	29.1	13.0	13.4
Method 1	112.1	126.3	18.8	14.9	4.2	–4.8	41.6	32.9	20.3	26.1
Method 3	107.8	107.8	23.6	21.9	18.7	–2.5	69.2	64.2	57.2	74.7
Method 4	108.8	105.5	19.5	18.5	8.3	–0.1	41.9	39.7	31.9	42.9

Table C5. Experiment 5; Model C; central basis, but with 40% equities and 60% index-linked

	$MF1$	$MF2$	$\sqrt{VF1}$	$\sqrt{VF2}$	$\sqrt{VF4}$	MC	$\sqrt{VC1}$	$\sqrt{VC2}$	$\sqrt{VC3}$	$\sqrt{VC4}$
Method 0	109.9	109.0	13.9	12.7	4.3	7.3	16.5	15.2	9.6	12.5
Method 1	110.2	109.4	13.9	12.7	4.4	7.2	17.7	16.2	11.2	14.8
Method 3	109.5	109.5	15.0	13.7	9.3	7.1	29.6	27.0	23.3	30.5
Method 4	108.1	100.9	14.2	14.1	4.8	8.9	16.6	16.5	12.1	16.3

Table C6. Experiment 6; Model C; central basis, but with 100% in index-linked bonds

	*MF*1	*MF*2	√*VF*1	√*VF*2	√*VF*4	*MC*	√*VC*1	√*VC*2	√*VC*3	√*VC*4
Method 0	96.4	89.3	8.9	10.0	4.4	18.2	5.4	6.0	2.6	2.8
Method 1	96.6	89.4	8.8	9.8	4.4	18.2	5.3	5.9	2.6	2.8
Method 3	102.2	102.2	4.8	4.7	1.8	18.0	5.4	5.3	2.4	2.5
Method 4	95.7	88.0	3.5	4.0	1.7	19.0	3.2	3.7	1.3	1.3

Table W7. Experiment 7; Model W; central basis, but with 60% in U.K. equities, 5% in cash, 10% in consols, 5% in index-linked and 20% in U.S. equities with 1-year currency forwards

	*MF*1	*MF*2	√*VF*1	√*VF*2	√*VF*4	*MC*	√*VC*1	√*VC*2	√*VC*3	√*VC*4
Method 0	110.8	99.7	15.7	15.7	9.5	2.6	28.6	28.6	23.3	31.4
Method 1	110.1	99.1	16.1	16.2	9.4	2.7	29.5	29.8	24.5	33.3
Method 3	109.3	109.3	20.9	19.2	19.5	–3.0	61.8	56.6	53.7	73.1
Method 4	107.3	90.7	16.3	18	10.1	4.6	26.9	29.7	24.0	33.6

Table W8. Experiment 8; Model W; central basis, but with 60% in U.K. equities, 5% in cash, 10% in consols, 5% in index-linked and 20% in U.S. equities without 1-year currency forwards

	*MF*1	*MF*2	√*VF*1	√*VF*2	√*VF*4	*MC*	√*VC*1	√*VC*2	√*VC*3	√*VC*4
Method 0	113.9	109.0	14.3	13.1	8.8	–1.2	29.6	27.1	22.3	29.6
Method 1	113.6	108.7	14.0	12.9	9.0	–1.1	32.3	29.7	25.5	34.8
Method 3	110.3	110.3	20.8	18.9	19.7	–4.8	63.2	57.3	55.1	74.9
Method 4	109.2	92.3	15.2	16.5	10.2	3.2	25.8	27.9	23.3	32.6

Table W9. Experiment 9; Model W; central basis, but with 80% in U.K. equities, 5% in cash, 10% in consols, 5% in index-linked and 0% in U.S. equities

	*MF*1	*MF*2	√*VF*1	√*VF*2	√*VF*4	*MC*	√*VC*1	√*VC*2	√*VC*3	√*VC*4
Method 0	110.6	102.4	14.8	14.4	7.3	4.1	24.0	23.4	17.5	22.6
Method 1	110.5	102.4	14.7	14.4	7.4	4.2	25.6	25.0	19.6	26.0
Method 3	108.5	108.5	20.8	19.2	19.8	–1.2	59.3	54.7	52.0	70.2
Method 4	106.1	89.7	15.7	17.5	9.9	6.5	24.9	27.7	22.8	30.8

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